Channel Management in Outpatient Care: Implications of Telemedicine and Transportation Support

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Abstract: The COVID-19 pandemic accelerated telemedicine adoption, offering a convenient alternative to in-person care. However, televisits may not fully address health concerns and sometimes require supplementary in-person visits, consuming resources that could have been saved if the initial visit had been in-person. As the pandemic subsides, in-person visits are regaining popularity, prompting providers to reorient resources toward in-person care. Transportation support (or subsidies) for patients, funded by providers or the government, plays a critical role in facilitating in-person visits. In this evolving landscape of telemedicine, we study how an outpatient care provider can optimally balance virtual and in-person services and whether, and how, to engage with transportation subsidies. We connect these two questions by examining how transportation subsidies reshape the provider's optimal capacity allocation across service channels and, in turn, affect overall patient access. We develop a stylized queueing-game model to represent the operations of a revenue-maximizing provider serving patients who strategically choose between service channels. We find that provider size, measured by total capacity relative to demand, is key. Small and large providers perform best by focusing on one channel without offering subsidies, whereas medium-sized providers benefit from carefully balancing both channels alongside subsidies. Paradoxically, transportation subsidies, which make in-person care more accessible, may reduce overall patient access to care, even when fully funded by the government. This occurs because providers may shift capacity toward a higher-reimbursement channel, ultimately serving fewer patients. Differentiating payment rates between in-person and virtual visits can potentially prevent such reductions. Our study highlights the importance of capacity coordination between channels for providers and cautions policymakers that transportation support may unintentionally harm patient access. Properly designed financial incentives can help prevent such negative outcomes.

Key words: healthcare operations management, telemedicine, queueing game, capacity management

1 Introduction

The COVID-19 pandemic accelerated the adoption of virtual services in general and telemedicine in the context of healthcare services in particular (Bokolo 2020). Telemedicine enables the provision of remote clinical services through real-time communication between patients and healthcare providers, utilizing tools like video conferencing and remote monitoring (Monaghesh and Hajizadeh 2020). Virtual services, which reduce travel costs and are associated with lower waiting costs for patients, can improve the efficiency of care delivery and expand access to care (Wong et al. 2021). However, they also pose a risk of delivering low-value care (O'Reilly-Jacob et al. 2021). Research has shown that a virtual visit may lead to a supplementary in-person visit for the same medical concern within a short timeframe, potentially due to misdiagnosis or inadequate treatment (Shi et al. 2018). Consequently, telemedicine may increase follow-up care and hurt provider productivity (Bavafa et al. 2018, Li et al. 2021). The evidences, however, are not always consistent. Other studies indicate that the increase in supplementary in-person visits may be minimal (Reed et al. 2021).

Despite these mixed results, telemedicine is expected to continue growing beyond the COVID-19 era, although it cannot fully replace in-person visits (Rosenthal 2021). In-person care provides benefits that virtual care cannot replicate, such as better communication and a stronger doctor-patient relationship. Moreover, certain diagnosis and tests can only be conducted in person.

During the pandemic, payment parity was introduced to encourage the adoption of telemedicine to expand care access, by reimbursing televisits at the same rate as in-person visits. With the pandemic now over, the downsides of telemedicine have drawn increased attention. A prevailing view is that reimbursing virtual visits—typically involving fewer diagnostic services—at the same rate as in-person visits represents overpayment. This has shifted policy discussions toward payment equity rather than parity in the post-pandemic era (Shachar et al. 2020). Meanwhile, in-person visits are regaining popularity among both patients and providers.

In response to these shifts, providers have started re-orienting their resources toward in-person visits. A 2022 McKinsey report highlights that as the pandemic abates, more physicians are gravitating away from virtual care and would prefer a return to in-person care (Cordine et al. 2022). There has been a 13% increase in physicians recommending in-person visits over telehealth and a 10% rise in physicians offering in-person care only since July 2020. Providers are making various efforts to encourage and support in-person visits, such as improving the in-clinic encounter experience, investing in a better physical service environment, and offering transportation support.

Transportation support, in particular, plays a critical role in facilitating in-person visits, especially for rural populations and those facing transportation challenges. It can be funded by the government. For example, federal Medicaid regulations (42 CFR 440.170) require that states ensure transportation for beneficiaries to and from providers (Medicaid and CHIP Payment and Access Commission 2019), a benefit known as non-emergency medical transportation (NEMT). Covered transportation modes include taxis, buses, vans, and personal vehicles. States have considerable flexibility in administering NEMT and may deliver the benefit via fee-for-service, managed care contracts, or transportation brokers.

Transportation may also be subsidized by providers through means such as travel reimbursement, shuttle services, coordinated shared ride programs, parking vouchers, or transit passes. For example, Family Health Services in Ohio and Kheir Clinic in Los Angeles both provide their own no-cost transportation services for patients; the Upper Allegheny Health System in southwestern New York and Bradford Regional Medical Center in northwestern Pennsylvania partner with the Area Transportation Authority of North Central

Pennsylvania to provide free transportation services (Graham 2022). In this paper, we use the term "transportation subsidy" broadly to refer to any support provided to help patients attend in-person visits.

Our research is motivated by this changing landscape of telemedicine, in which all stakeholders are rethinking how best to integrate virtual care into the broader care delivery system. In particular, we seek to understand the interplay between telemedicine and transportation support, and its implications for care delivery. We focus on non-emergency outpatient care settings, and investigate how providers can optimally balance virtual and in-person services, and whether and how they should engage with transportation subsidies. Our study also sheds light on how provider operational choices impact access to care.

When seeking outpatient care, patients are typically presented with two options—coming in for an in-person visit or having a virtual visit. Patients weigh the utilities and costs associated with these alternatives before deciding. In-person visits require waiting in the clinic, incurring both time spent and potential travel costs (e.g., transportation and parking). Conversely, virtual visits offer the convenience of engaging from home or another preferred location by patients, thus reducing their waiting and travel costs. For some health conditions, a virtual visit can fully address patients' concerns. However, in other cases, a supplementary in-person visit (or returning visit) may be required if the initial virtual treatment is inadequate or symptoms do not improve. In such scenarios, patients need to subsequently visit the clinic, incurring the costs and disutilities initially avoided.

Facing strategic patient choice, an outpatient care provider needs to determine the optimal system design and capacity allocation between in-person and virtual channels. Whereas the virtual channel is appealing for its convenience to both providers and patients, it may lead to returning in-person visits, which come with additional cost and capacity utilization. To support in-person visits, mitigate the need for returning visits, and improve overall operational efficiency, transportation subsidies may be offered, as discussed above. Though offering transportation subsidies can be useful to improve a provider's operational efficiency and revenue, its impact on patient access to care is unclear. Intuitively, transportation subsidies should expand patient access to care because they make in-person care more accessible, but does this always hold true?

To answer our research questions, we develop a stylized queueing-game model to represent the operations of an outpatient care provider, who has a fixed daily capacity to allocate between two channels. The provider faces an exogenous stream of strategic patient demand. Each patient evaluates the expected waiting in each channel and the likelihood of needing a returning in-person visit, with three options available: choose an in-person visit, wait for a virtual visit, or balk. Balking encapsulates other care alternatives the patient may choose, such as visiting other care providers or taking some at-home treatments.

If an in-person visit is chosen, the patient's expected utility is the service reward net the expected waiting cost and travel cost associated with the in-person channel. If a virtual visit is chosen, the patient's expected utility incorporates the expected waiting cost in the virtual channel and the expected utility of a returning visit, which will incur additional waiting and travel costs to attend the in-person visit. Lastly, if the patient balks, the utility is normalized to be zero. Importantly, patient choice is endogenous to the provider's capacity allocation because waiting cost depends on service capacity allocation between the two channels. Factoring into patient strategic choice, the provider aims to maximize the total revenue across both service channels by carefully allocating her daily capacity.

We first show that for any given capacity allocation, there exists a unique mixed-strategy equilibrium in patient choice. With three alternatives available (virtual visit, in-person visit, or balking), there are seven equilibrium regions based on how patients mix their choices. We then fully characterize the optimal capacity allocation strategy for the provider to maximize her revenue. The optimization incorporates patient equilibrium, which is endogenous to the capacity allocation. We find that the optimal system configuration (i.e., how much capacity to allocate to each channel) depends on the size of the system, measured by the total available service capacity relative to the overall patient demand. In a small system, the provider should pool her capacity to focus on one service channel to maximize efficiency. For a medium-sized system, she has enough capacity to prevent patient balking, but needs to carefully balance capacity between the two channels. For a large system, because no patients would balk regardless of the capacity allocation, a revenue-maximizing provider should prioritize the channel with the highest revenue margin.

Next, we consider the setting with transportation subsidies. We differentiate two scenarios: subsidies paid by the provider and subsidies funded by the government. For the first case, we fully characterize the joint optimal decision for capacity allocation and subsidy levels. For the second case, because the subsidies incur no cost to the provider, the decision focuses solely on capacity allocation. It is natural to expect that the use of transportation subsidy increases total revenue, regardless of who pays for it. What is more interesting is that the optimal system design can fundamentally change with the subsidies offered. For example, a system optimally designed to focus on the virtual channel without subsidies may be optimized by changing to the in-person channel with subsidies, or vice versa. Intuitively, transportation subsidies should increase the total rate at which patients access the service across both channels because subsidies can attract more in-person visits, which do not require returning visits and thus conserve service resources. However, our results show this is *not* always the case: offering subsidies can paradoxically reduce the total access rate, even when funded entirely by the government at no cost to the provider!

Our numerical analysis, calibrated with real-world data and empirical studies, confirms this "backfire" phenomenon. The underlying cause is the potential shift of the provider's optimal strategy: she may choose to serve fewer patients at a higher payment rate, leading to a decrease in total access rate. To mitigate such an adverse impact on social welfare, one solution we identify is in line with the idea of "payment equity" discussed above. Specifically, we show that differentiating payment rates between in-person and virtual visits can prevent reductions in access rate, regardless of who funds transportation subsidies. Our results, at a high level, also support the current Centers for Medicare & Medicaid Services (CMS) policy that grants states flexibility in administering NEMT, as this flexibility allows states to tailor transportation support in a way that aligns with provider incentives and care delivery goals to help prevent unintended reductions in access to care.

The rest of the paper is organized as follows. Section 2 reviews the relevant literature. Section 3 presents the model and analyzes patient equilibrium strategy. Section 4 studies the provider's optimal capacity allocation without transportation subsidies. Section 5 extends the analysis to the setting with subsidies. Section 6 conducts analytical and numerical comparisons between provider- and government-funded subsidies. Section 7 concludes the paper by summarizing the key managerial and policy insights and suggesting directions for future research. All technical proofs are provided in the E-Companion.

2 Literature Review

This research draws upon the literature on healthcare operations management (OM) and queueing studies with strategic customers and multiple service access channels. We review each stream below.

2.1 Healthcare OM

Within the healthcare OM literature, our work is related to the research that (1) employs queueing models to investigate system design questions in outpatient care, and (2) addresses the multi-channel care setting, in particular those with one telemedicine channel. Two representative studies from the first stream of literature include Green and Savin (2008) and Zacharias and Armony (2016). They develop stylized queueing models to investigate system design questions such as panel size selection and capacity decisions. An important phenomenon addressed by these two works is that patients may revisit the provider after (no-shows from) initial visits. Though this patient "returning" feature also presents in our model, our patient revisits require a different source of capacity compared to their initial visits. Moreover, patient demand in Green and Savin (2008) and Zacharias and Armony (2016) are treated as exogenous variables, whereas we model patients as strategic customers, rendering patient demand endogenous to the provider's decisions.

Depending on the clinical setting, patients may access care through different channels, e.g., in-person appointment visits, walk-ins, virtual visits, or visits to different providers. These channels can be managed by a single provider or coordinated among/independently offered by multiple providers. The literature addressing OM issues in such multi-channel care settings is growing. Liu et al. (2023) examine capacity management between appointment-based and walk-in services within a single provider setting. In their model, only strategic walk-in patients may balk, and patients are assumed to fully recover regardless of the chosen channel. In multi-provider settings, Shumsky and Pinker (2003) introduce a classic gatekeeper model to study gatekeeper-specialist dynamics. A more recent work by Sharma et al. (2020) focuses on the interaction between emergency rooms and general providers. Both of these studies explore incentive design, aiming to align different providers who offer in-person service towards societal or firm-centric objectives. On a separate topic, Huang et al. (2022) study the doctor-shopping behavior (i.e., patients seek opinions from multiple doctors without referrals) and its impact on social welfare. In contrast to the latter three studies on multi-provider settings, our research concentrates on the operations of a single provider who offers both in-person and virtual services.

With the rise of telemedicine, the body of literature assessing its operational impact is expanding. Rajan et al. (2019) and Bavafa et al. (2021) study the use of telehealth visits in chronic care. The former finds that telemedicine can improve access to care and increase provider revenue, whereas the latter shows that e-visits may negatively impact panel size and patient health. Tele-triage is another form of telemedicine, aimed at remotely assessing patient conditions to determine the appropriate care they need. Çakıcı and Mills (2021) and Guan et al. (2025) evaluate the use of tele-triage, and discover that it may increase cost and exacerbate inefficiencies in a healthcare system. In contrast to these studies, we focus on how an outpatient care provider can effectively integrate virtual visits into her operations.

A few recent studies have explored a hybrid care delivery model similar to ours, involving in-person and virtual channels with returning visits (or, more generally, imperfect treatment). Focusing on the Primary Care First (PCF) initiative, Adida and Bravo (2023) analyze blends capitation and fee-for-service payments with performance-based adjustments to incentivize redesigned primary care delivery, including remote care. They find that PCF can achieve socially optimal outcomes by adjusting payment structures based on state-specific health heterogeneity.

The most relevant work to ours is Çakıcı and Mills (2024), which investigates the repercussions of telehealth reimbursement policies on the accessibility of acute care. They show that returning visits resulting from virtual care create an incentive alignment problem, and thus pay parity for telehealth may hurt patient access. Our study departs from theirs in at least two significant ways. First, our base model without transportation subsidies *fully* characterizes the provider's optimal capacity allocation decisions under *all* possible demand-capacity scenarios, whereas Çakıcı and Mills (2024) focuses only on a high-workload setting. Second, in contrast to their focus on incentive structures designed during the pandemic to promote telemedicine adoption, we examine issues that are increasingly salient in the post-pandemic era: specifically, whether a hybrid care delivery model combining in-person and virtual services improves patient access to care, particularly in the presence of transportation subsidies.

In addition, Zychlinski (2024) tackles scheduling and capacity allocation decisions across three service channels: in-person, virtual, and supplementary service for returning virtual patients. Zang et al. (2024) explore how improving patient understanding of their telemedicine suitability through online triage affects the performance of a hybrid system. Wang et al. (2019) study dual-channel care (in-person and virtual) where some patients receiving virtual care require returning in-person visits. Their model concerns two independent providers who set their capacity separately in a capacity-abundant environment, whereas we study how a single provider should operate under any capacity situation. Complementing these recent studies, our work contributes to a more comprehensive understanding of how to effectively manage hybrid healthcare systems that incorporate telemedicine.

2.2 Queueing Studies with Strategic Customers

In terms of methodology, our research leverages the analysis of queuing systems with strategic customers. This broad research area, initiated by Naor (1969), explores customer join/balk decisions based on how sensitive customers are to waiting and aims to optimize system efficiency/social welfare by adjusting the service capacity, pricing, or priority schemes. Hassin and Haviv (2003) provide a comprehensive review of this area. Below we draw attention to recent studies most relevant to our work.

Hassin and Roet-Green (2020) study the impact of queue-length information on the performance of a service system where customers must travel to join. In their model, customers can observe the queue length before deciding whether to travel. Another recent relevant paper that considers travel costs is by Baron et al. (2022). They examine an omni-channel service system that offers both walk-in and online channels. To receive the final service, customers incur travel costs regardless of the channel chosen. They show that although the online channel increases revenue, in equilibrium it reduces customer utility and social welfare. They propose prioritizing walk-in customers to mitigate these effects. In contrast to these two works, we consider a healthcare setting that offers a virtual service channel with no travel costs and an in-person channel that requires travel for both initial and returning visits. We focus on capacity rather than (scheduling) priority decisions.

2.3 Our Contribution

We conclude this section by briefly summarizing our contributions. We develop a modeling framework for an outpatient care provider who offers both in-person and virtual services. Our model captures the key trade-offs that strategic patients face when choosing between these two channels. A notable feature of our model is that it captures the difference in care quality between channels: patients who choose the virtual channel may need an in-person return visit. In the post-pandemic era, in-person visits are regaining popularity, and transportation support is one of the most effective ways to facilitate these visits, either offered by the provider or subsidized by the government. We also study how transportation support can be incorporated into provider operations and its broader impact.

In our analysis, we first prove the existence and uniqueness of a mixed strategy equilibrium for patient choice, based on which we characterize the optimal capacity management strategy for a revenue-maximizing provider. We find that the size of the system, measured by the total available service capacity relative to the overall patient demand, plays a critical role in determining the optimal capacity allocation of the provider. Small and large systems are better off focusing on one channel and do not need to offer transportation support, whereas medium-sized systems can benefit from a careful balance between the two channels alongside transportation support. However, we caution that providing transportation support can backfire and reduce overall patient access to care, even when fully funded by the government at no cost to the provider. To prevent such outcomes, one solution we identify is to differentiate payment rates between in-person and virtual visits.

3 The Model

We consider an outpatient care provider offering two service channels: in person and virtual, with a fixed daily capacity of μ. The provider decides how to allocate the in-person capacity (μ_f) and the virtual capacity (μ_v) such that $\mu_f + \mu_v = \mu$. The actual service time of patients may have some variability, but with an allocation (μ_f, μ_v), the provider is expected to be able to serve μ_f in-person patients and μ_v virtual patients per day. In practice, outpatient care providers often reserve specific slots in their daily appointment book for in-person and virtual visits, respectively. Our capacity allocation model reflects this common approach.

Homogeneous strategic patients arrive following a Poisson process with daily rate Λ. Each patient, upon his[1] arrival, strategically chooses between in-person and virtual service channels, if both are offered by the provider. Some patients may need a supplementary in-person visit with the same provider after the virtual visit. This could be due to the challenges of virtual diagnosis (as with some skin conditions), the need for additional tests before prescribing medication (e.g., urinary tract infection), or the lack of improvement in patient symptoms with at-home treatments (such as in cases of persistent sore throats). We assume that for each patient, the *ex ante* probability of requiring a supplementary visit is δ, and a smaller δ indicates greater effectiveness of virtual care. We also assume that in-person visits always resolve patient health concerns, so patients do not need to revisit the provider after an in-person visit for the same issue. Note that supplementary visits resulting from the ineffectiveness of virtual care are fundamentally different from follow-up in-person visits triggered by other health concerns, e.g., chronic conditions. We use the term "supplementary" to emphasize that this visit addresses the same episode of health issue that was not fully resolved by the initial virtual visit. In contrast, follow-up visits are not the result of an ineffective initial visit but are often required by clinical protocols. We assume that the demand for such follow-up visits depends solely on population characteristics and can therefore be incorporated into the daily demand rate Λ.

Moreover, patients may have a range of other care options, such as seeking care at emergency departments, considering urgent care service, or resorting to self-care at home (Khairat et al. 2021). To encapsulate all these options, we include "balking" as a third option within the patient's choice set. This approach allows us to capture the full range of patient decisions in response to the offered services. Patients make their choices based on the expected utilities of these options, which depend on the expected wait time to get service, the associated travel cost, if any, as well as the need for a supplementary visit.

Given that patients are *ex ante* homogeneous, we identify the equilibrium in the class of mixed and symmetric strategies. Denote any mixed strategy by (p_f, p_v, p_b), where p_f, p_v, and p_b, respectively, represent the probabilities that a random patient, upon his arrival, chooses the in-person channel, the virtual channel, and balking. Note that $p_f + p_v + p_b = 1$ because these three options are exhaustive. Hence, we can simplify the representation of the mixed strategy as (p_f, p_v). Let $\lambda_f = p_f\Lambda$ and $\lambda_v = p_v\Lambda$ denote the respective arrival

[1] For convenience, we shall refer to the provider as "she" and a patient as "he" in the rest of this article.

rate to the in-person channel and virtual channel. Then, the pair (λ_f, λ_v) represents the effective arrival rate to each channel when a mixed strategy (p_f, p_v) is adopted. Given that Λ is fixed, we also use (λ_f, λ_v) to denote the patient strategy for notational simplicity.

Our model can be extended to incorporate patient heterogeneity in travel cost and health needs (see Remark 2 and Remark 4, respectively). However, since patient heterogeneity does not alter the key insights, we focus on the current model with homogeneous patients for clarity and ease of discussion.

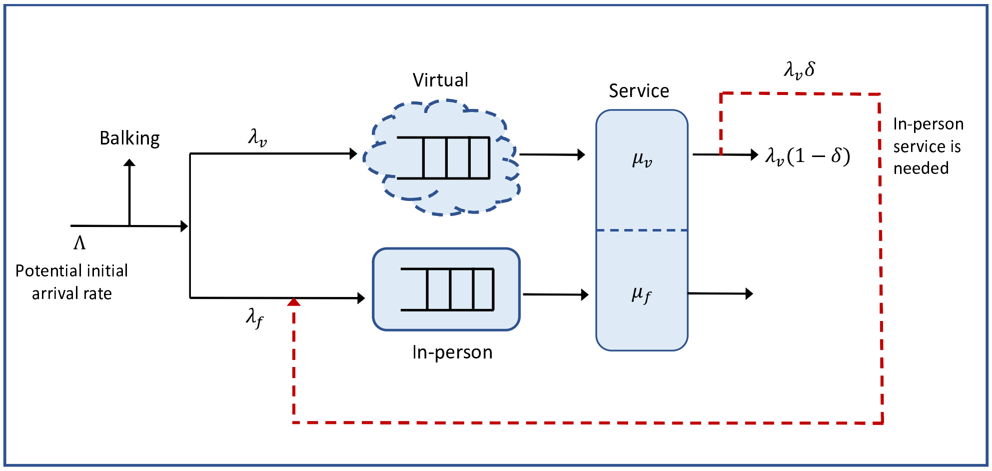
3.1 Patient's Utility

To analyze patient strategy, we start by examining the utility of each choice. We begin with the arrival–service process under any given strategy (λ_f, λ_v). Recall that the provider divides her daily capacity into in-person service slots and virtual slots, so that she has the capacity to serve μ_f in-person patients and μ_v virtual patients per day. In-person (virtual) slots are dedicated to in-person (virtual) patients only. Thus, one can view the in-person and virtual service channels as two independent "servers".

Inspired by prior literature employing stylized single-server queueing models to address strategic-level questions in outpatient care services (Green and Savin 2008, Liu and Ziya 2014, Zacharias and Armony 2016, Liu et al. 2023), we adopt a network of two stylized single-server queues to capture the evolution of system dynamics. When a patient requests a virtual visit, he is scheduled to the end of the virtual care queue (i.e., added to the virtual visit appointment backlog). Upon completion of virtual service, if a supplementary in-person visit is required, the patient is scheduled to the end of the in-person care queue. Similarly, a patient requesting an in-person visit is directly added to the end of the in-person care queue (i.e., to the in-person visit appointment backlog). In this framework, queue waiting encompasses both appointment delay and waiting in the actual or virtual service line for care. For tractability, and without losing key insights, we model both the arrival and service processes as Markovian, following Green and Savin (2008), Liu and Ziya (2014) and Liu (2016). Hence, this queueing network is a Jackson network (see Figure 1). In steady state, it functions as two M/M/1 queues, one representing the virtual channel and the other the in-person channel. The arrival and service rates are λ_v and μ_v, respectively, in the virtual queue, and $\lambda_f + \delta\lambda_v$ and μ_f, respectively, in the in-person queue.

Transportation Subsidies. Transportation service is an important support mechanism for in-person visits. It may be financed directly by the provider or subsidized by the government, in which case it is cost-free to the provider. Transportation support can take various forms, including reimbursement of travel expenses, shuttle services, coordinated ride programs, or public transit passes. Consistent with our central focus on capacity design, we examine how transportation subsidies reshape the provider's optimal capacity allocation and, in turn, affect patient access to care. For ease of exposition, we use the term "subsidy" as an umbrella term encompassing these different forms of transportation support.

Figure 1 Illustration of the model featuring patients choosing among virtual visit, in-person visit or balking.



The impact of transportation subsidies on patient behavior is multifaceted. On the one hand, a subsidy may encourage patients to select the in-person channel by reducing their effective travel cost. On the other hand, it may make the virtual channel more appealing, as patients anticipate that a follow-up in-person visit——if required——would be subsidized. We let $b \geq 0$ denote the transportation subsidy for each in-person visit. (Although interpreted as a transportation subsidy, b more generally captures any intervention that reduces patients' (perceived) cost of accessing in-person care, such as reduced co-payments, childcare support, and parking vouchers. This broader interpretation extends the applicability of our model beyond transportation-focused policies.)

Next, we quantify the expected utility associated with each option.

Cost of each in-person visit. If a patient chooses the in-person channel, he incurs a cost T, which represents the disutility associated with each in-person visit, such as inconvenience and cost due to transportation. For ease of discussion, We call T the travel cost. Since each queue in the Jackson network can be regarded as an M/M/1 queue, the expected waiting time in the system for each in-person visit is

$$W_f(\lambda_f, \lambda_v) = \frac{1}{\mu_f - (\lambda_f + \delta\lambda_v)}. \tag{1}$$

Let θ_f denote the waiting cost per unit of time in the in-person channel. Then, the expected total cost associated with each in-person visit is

$$C_f(\lambda_f, \lambda_v) = T - b + \theta_f W_f(\lambda_f, \lambda_v). \tag{2}$$

Cost of each virtual visit. The expected waiting time in the system for the virtual channel is

$$W_v(\lambda_v) = \frac{1}{\mu_v - \lambda_v}. \tag{3}$$

Let θ_v denote the waiting cost per unit of time for patients in this channel. Since patients choosing the in-person channel must commute to and stay in the clinic, whereas patients on the virtual channel can see providers in a more comfortable environment such as their homes, it is natural to impose that $\theta_f \geq \theta_v$.

Though the virtual channel incurs no travel cost and has a lower waiting cost rate, patients starting with the virtual channel may need a supplementary in-person visit, with all the additional disutility associated with it. The expected total cost of each virtual visit (followed by the necessary in-person visit) is, therefore,

$$C_v(\lambda_f, \lambda_v) = \theta_v W_v(\lambda_v) + \delta C_f(\lambda_f, \lambda_v). \tag{4}$$

Utility of each option. Patients receive a service reward R once their health concerns are addressed. Without loss of generality, we normalize the utility of balking to be 0. Let $U_f(\cdot)$, $U_v(\cdot)$, and $U_b(\cdot)$ represent the expected utilities of choosing in-person visit, virtual visit, and balking, respectively. Regardless of who pays for the subsidy, by (1)–(4), we have the following expressions:

$$\begin{aligned} U_f(\lambda_f, \lambda_v) &= R - T + b - \frac{\theta_f}{\mu_f - \lambda_f - \delta\lambda_v}; \\ U_v(\lambda_f, \lambda_v) &= R - \frac{\theta_v}{\mu_v - \lambda_v} - \delta T + \delta b - \frac{\delta\theta_f}{\mu_f - \lambda_f - \delta\lambda_v}. \\ U_b(\lambda_f, \lambda_v) &= 0. \end{aligned} \tag{5}$$

REMARK 1 (INCORPORATING PATIENT OUT-OF-POCKET PAYMENTS). In practice, outpatient visits are typically covered by insurance, and patients pay only a modest co-payment or other out-of-pocket expenses. These expenditures are largely independent of the provider's operational decisions. While out-of-pocket amounts may differ between the in-person and virtual channels, they are treated as exogenous to the model.

Let p_f denote the out-of-pocket payment for the in-person channel and p_v that for the virtual channel. Define the adjusted parameters for service reward and travel cost as follows:

$$R' = R - \frac{p_v - \delta p_f}{1 - \delta} \quad \text{and} \quad T' = T + p_f - \frac{p_v - \delta p_f}{1 - \delta}.$$

It is straightforward to verify that

$$R - p_f - T = R' - T' \quad \text{and} \quad R - p_v - \delta T = R' - \delta T'.$$

Substituting these expressions into the utility functions $U_f(\lambda_f, \lambda_v)$ and $U_v(\lambda_f, \lambda_v)$ in (5) shows that, for any given subsidy level $b \geq 0$, the utilities retain the same functional forms under the adjusted parameters (R', T'). Consequently, all analytical results derived in this paper remain valid when accounting for patient out-of-pocket payments.

3.2 Patient Equilibrium Strategy

Given (μ_f, μ_v), a strategy (λ_f, λ_v) is a symmetric equilibrium strategy if it constitutes the best response to itself. In other words, no patient can unilaterally increase his own utility by deviating from this strategy. More formally, the equilibrium effective arrival rates (λ_f, λ_v) must satisfy the following condition:

CONDITION 1. For $x = f, v$ or b, if $\lambda_x > 0$, then we have

$$U_x(\lambda_f, \lambda_v) = \max\{U_f(\lambda_f, \lambda_v), U_v(\lambda_f, \lambda_v), U_b(\lambda_f, \lambda_v)\}.$$

According to Condition 1, Table 1 provides a summary of the different strategies along with the conditions on their corresponding effective arrival rates and utilities. To illustrate various strategies, we use the following symbols: B for the pure strategy of balking, V for the pure strategy of choosing the virtual channel, and F for the pure strategy of selecting the face-to-face channel (i.e., the in-person channel). The mixed strategies, combining these choices, are denoted by the letter combinations of B, V, and F. Altogether, there are seven distinct types of strategies: B, V, F, BVF, BF, BV and VF. Note that Strategy B, in which all patients balk, is included only for analytical completeness and is not emphasized as a realistic operational regime.

Table 1 Equilibrium Strategies

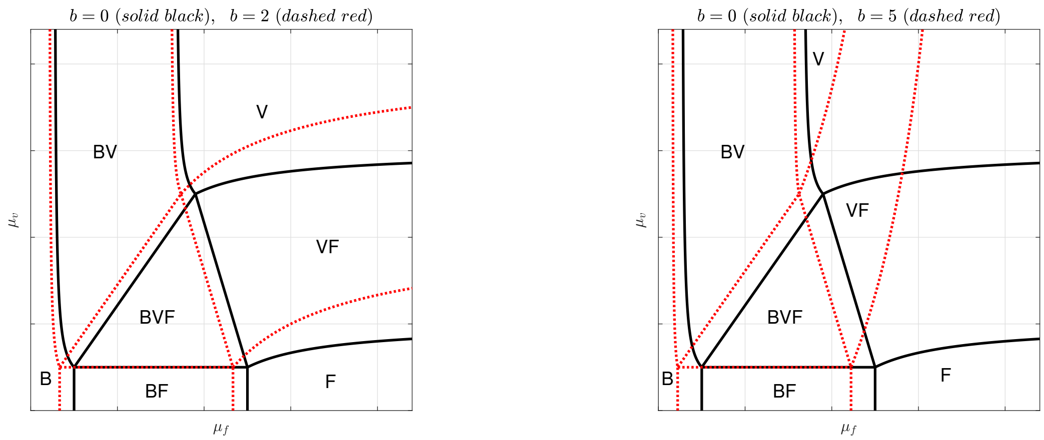
Strategy	Effective Arrival Rates	Utilities
B	$\lambda_f = 0,\ \lambda_v = 0$	$U_f(0,0) \le 0,\ U_v(0,0) \le 0$
V	$\lambda_f = 0,\ \lambda_v = \Lambda$	$U_f(0,\Lambda) \le U_v(0,\Lambda),\ U_v(0,\Lambda) \ge 0$
F	$\lambda_f = \Lambda,\ \lambda_v = 0$	$U_f(\Lambda,0) \ge U_v(\Lambda,0),\ U_f(\Lambda,0) \ge 0$
BVF	$\lambda_f + \lambda_v \le \Lambda$	$U_f(\lambda_f,\lambda_v) = U_v(\lambda_f,\lambda_v) = 0$
BF	$\lambda_f \le \Lambda,\ \lambda_v = 0$	$U_f(\lambda_f,0) = 0,\ U_v(\lambda_f,0) \le 0$
BV	$\lambda_f = 0,\ \lambda_v \le \Lambda$	$U_f(0,\lambda_v) \le 0,\ U_v(0,\lambda_v) = 0$
VF	$\lambda_f + \lambda_v = \Lambda$	$U_f(\lambda_f,\lambda_v) = U_v(\lambda_f,\lambda_v) \ge 0$

Following Table 1, we derive the explicit conditions on (μ_f, μ_v) that determine which strategy is adopted. These conditions define seven regions for service capacity allocation, namely Region B, V, F, BVF, BF, BV and VF. For instance, Region BVF encompasses capacity allocation (μ_f, μ_v), where patients choose the BVF strategy, and is described by the following inequalities:

$$\mu_v \ge \frac{\theta_v}{(1-\delta)R},$$
$$\mu_f - \delta\mu_v \ge \frac{\theta_f}{R-T+b} - \frac{\delta\theta_v}{(1-\delta)R}, \text{ and}$$
$$\mu_f + (1-\delta)\mu_v \le \Lambda + \frac{\theta_f}{R-T+b} + \frac{\theta_v}{R}.$$

The specific closed-form definitions for all regions can be found in Definition EC.1 of the E-Companion. With the x-axis representing μ_f and the y-axis representing μ_v, Figure 2 depicts these seven regions when $b = 0$ (solid lines) and for two cases in which $b > 0$ (dotted lines). Note that when $b > 0$, the equilibrium strategies exhibit a structure similar to the case without a subsidy. However, the partition into the seven regions may change as a function of b. In particular, the sign of $(T-b)$ affects the shapes of the boundaries between Regions V and VF, as well as between Regions F and VF.

Figure 2 Illustrations of equilibrium regions under different values of b.



Note. $\Lambda=1$, $T=4$, $R=8$, $\delta=0.7$, $\theta_f=1$, $\theta_v=0.6$.

We observe that with a positive transportation subsidy b, the boundaries shift to the left, leading to a smaller Region B. When $T-b$ is positive (as shown in the left plot), the boundary shapes remain unchanged. In contrast, when $T-b$ is negative (as shown in the right plot), the shapes change: Region V, where all patients choose the virtual channel, shrinks significantly, while Region F, where all patients choose the in-person channel, expands. Overall, offering a transportation subsidy for in-person visits reduces balking and has the potential to increase service utilization, particularly in the in-person channel.

Together with Definition EC.1, Theorem 1 formally establishes the existence and uniqueness of patient equilibrium for any given capacity allocation (μ_f,μ_v). The detailed equilibrium expressions for each region are presented in Theorem EC.1 of the E-Companion.

THEOREM 1 **(Existence and Uniqueness of Patient Equilibrium)**. *For any given capacity allocation (μ_f,μ_v), a patient equilibrium exists and it is unique. Specifically, the equilibrium strategies partition the (μ_f,μ_v) space into seven mutually exclusive and collectively exhaustive regions, each corresponding to a distinct combination of patient behaviors (balking, virtual, and in-person participation).*

REMARK 2 (INCORPORATING HETEROGENEOUS TRAVEL COSTS). In the current model, all patients have the same travel cost T. The model can be extended to incorporate heterogeneous (uniformly distributed) travel costs. The resulting patient equilibrium has a similar structure to that described in Theorem EC.1; see E-Companion EC.2 for details.

In equilibrium, when μ_f is small, no patient will choose the in-person channel; when μ_v is small, no patient will choose the virtual channel. When both μ_f and μ_v are small, all patients will balk. It is worth noting that μ_f impacts the utilities of both channels—in-person and virtual. In particular, when μ_f is small, those who choose virtual visits can still suffer because some of them need a supplementary in-person visit for which they will have to wait a long time and, accordingly, have low utility. Hence, when μ_f is small, regardless of how large μ_v is, some patients will balk. When μ_f is large, however, regardless of how small μ_v is, no patient will balk. To the best of our knowledge, Theorem 1 (and its detailed version, Theorem EC.1) is the first in the literature to *fully* characterize patient equilibrium strategies across *all* possible provider capacity allocations in this class of models. This characterization is essential for deriving the global optimal solution to the provider's subsequent optimization problem.

4 Optimal Capacity Allocation Without Subsidies

In this section, we study the provider's optimal capacity allocation under strategic patient behavior. We first analyze the benchmark case without transportation subsidies ($b = 0$). In Section 5, we extend the analysis to the case with $b > 0$ and examine both provider- and government-funded transportation subsidies.

Considering patient strategic behavior, the provider aims to maximize her expected daily rewards by choosing a capacity allocation (μ_f, μ_v) such that $\mu_f + \mu_v = \mu$. The provider receives payment r_f and r_v for each patient who chooses the in-person channel and the virtual channel, respectively. The provider's problem can be formally defined as follows.

$$\begin{aligned} \max_{\mu_f,\ \mu_v \geq 0} \quad & r_f \lambda_f + r_v \lambda_v \\ s.t. \quad & \mu_f + \mu_v = \mu, \\ & (\lambda_f, \lambda_v) \text{ is defined in Theorem EC.1.} \end{aligned} \tag{P}$$

For any given capacity allocation (μ_f, μ_v) in the feasible solution space, Theorem EC.1 confirms that there exists a unique patient equilibrium, and hence Problem (P) is well defined.

LEMMA 1 **(Relaxing the Capacity Constraint)**. *The optimal solution to* (P) *remains optimal if the capacity constraint $\mu_f + \mu_v = \mu$ is relaxed to $\mu_f + \mu_v \leq \mu$.*

REMARK 3 (FLEXIBLE REVENUE STRUCTURE). The provider's revenue structure in our model is quite flexible and can capture commonly-adopted reimbursement regimes in healthcare. The pair (r_f, r_v) naturally represents a bundle payment mechanism because r_v covers the whole episode of care, which may involve a supplementary in-person visit. Under the fee-for-service payment mechanism, where r_v^{FFS} and r_f^{FFS} represent the payments for each virtual and in-person visit respectively, the model can still be applied by setting the provider's rewards (r_f, r_v) as $(r_f^{\text{FFS}}, r_v^{\text{FFS}} + \delta r_f^{\text{FFS}})$.

Next, we investigate the optimal capacity allocation in problem (P). Recall that Definition EC.1 and Theorem EC.1 partition the solution space for capacity allocation (μ_f, μ_v) into seven regions. We start by exploring the optimal capacity allocation in each region. Specifically, for any Region X defined in Definition EC.1, we incorporate the constraints of (μ_f, μ_v) into Problem (P), where X=B, V, F, BVF, BF, BV or VF. We denote the optimal solution and the corresponding effective arrival rates subject to Region X, if they exist, as $\left(\mu_f^{X*}, \mu_v^{X*}\right)$ and $\left(\lambda_f^{X*}, \lambda_v^{X*}\right)$. Then, we compare the optimal objective values of (P) across all regions to find the global optimal capacity allocation.

Regions B,V and F are trivial cases where any feasible capacity allocation results in the same equilibrium. In Region BF, the equilibrium arrival rate is $\lambda_f^{BF} = \mu_f - \theta_f[R-T]^{-1}$ with $\lambda_v^{BF} = 0$; thus, the provider's revenue increases monotonically with μ_f, implying that all available capacity should be devoted to the in-person channel. The optimal allocations and equilibrium arrival rates for these straightforward regions are summarized in Proposition EC.1.

For Region BVF, because λ_v^{BVF} (λ_f^{BVF}) increases linearly in μ_v (μ_f) while other parameters are fixed, the optimal capacity allocation exhibits a bang-bang pattern: capacity should be concentrated entirely in the channel with the higher marginal revenue. This intuition is formalized in Proposition EC.2.

In Region BV, the equilibrium structure is more intricate since patient participation and balking coexist. Depending on the system capacity and payment rates, Region BV may dominate, coincide with, or be dominated by neighboring regions V and BVF. The exact boundaries and optimal allocations are derived in Proposition EC.3.

Finally, Region VF represents the primary operational regime where both channels are active. Here, the optimal allocation depends on the relative payment rates: if $r_f > r_v$, more capacity should be allocated to in-person service, whereas if $r_v > r_f$, the provider should favor virtual service. Proposition EC.4 provides the detailed derivations.

Building on the regional optimality results, we next determine the global optimal capacity allocation. We begin by introducing the system-size categories that will guide the subsequent analysis.

DEFINITION 1. Given the system's primitives, we consider the following system-size categories with respect to the total service capacity μ.

1. **Small System:**
$$\max\left\{\frac{\theta_f}{R-T}, \underline{\mu}^{BV}\right\} \leq \mu < \Lambda + \frac{\theta_f}{R-T} + \frac{\theta_v}{(1-\delta)R},$$
where $\underline{\mu}^{BV}$ denotes the minimum system capacity necessary to attain Region BV, as formally defined in Definition EC.2 of the E-companion.

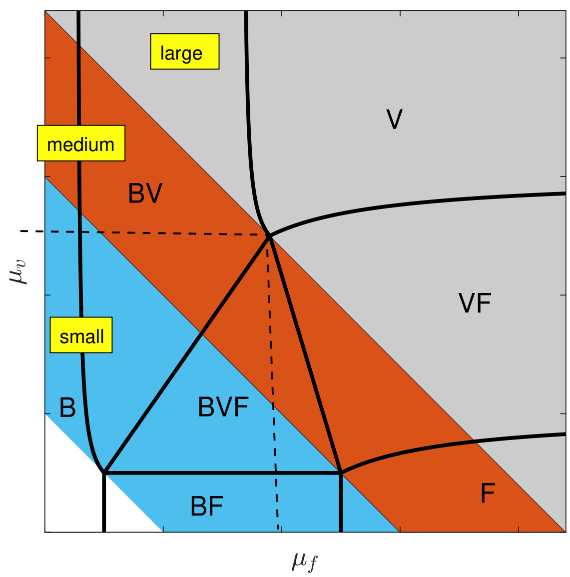
2. **Medium-sized System:**
$$\Lambda + \frac{\theta_f}{R-T} + \frac{\theta_v}{(1-\delta)R} \leq \mu < (1+\delta)\Lambda + \frac{\theta_f}{R-T} + \frac{\theta_v}{(1-\delta)R}.$$

3. **Large System:**

$$\mu \geq (1+\delta)\Lambda + \frac{\theta_f}{R-T} + \frac{\theta_v}{(1-\delta)R}.$$

Figure 3 illustrates the three system-size categories in different colors. The region where $\mu < \max\{\theta_f [R-T]^{-1}, \underline{\mu}^{BV}\}$, shown as the uncolored bottom-left corner, is not considered, because it represents very small systems in which patients either balk entirely or mix between balking and a single service channel.

Figure 3 Illustration of three system-size categories.



THEOREM 2 (**Optimal Capacity Allocation**). *The optimal capacity allocation (μ_f^*, μ_v^*) for each system-size category can be described as follows.*

1. ***Small System:** If $r_f \min\{\Lambda, \mu - \theta_f [R-T]^{-1}\} \geq r_v \lambda_v^{BV*}$, (μ_f^*, μ_v^*) is in Region BF (with Region F considered a special case of Region BF); otherwise, (μ_f^*, μ_v^*) is in Region BV.*

2. ***Medium-sized Systems:** If $r_v \lambda_v^{BV*} \geq r_f \Lambda + \delta^{-1}(r_v - r_f)^+(\mu - \Lambda - \theta_v [R(1-\delta)]^{-1} - \theta_f [R-T]^{-1})$, (μ_f^*, μ_v^*) is in Region BV; otherwise, (μ_f^*, μ_v^*) is in Region VF (with Region F considered a special case of Region VF and dominating Region VF when $r_f \geq r_v$).*

3. ***Large System:** If $r_v \geq r_f$, (μ_f^*, μ_v^*) is in Region V; otherwise, (μ_f^*, μ_v^*) is in Region F.*

REMARK 4 (INCORPORATING HETEROGENEOUS HEALTH NEEDS). The current model assumes that all patients can initially be served by either the in-person or virtual channel. The model can be extended to incorporate heterogeneous health needs as follows: some patients, arriving at an exogenous rate of λ_E, must access the in-person channel. The provider "carves out" μ_E service capacity for them and then allocates the remaining $\mu - \mu_E$ capacity between the in-person and virtual channels for strategic patients who choose between the two. All results in Sections 3 and 4 directly apply to assist the provider in making capacity allocation decisions in this case.

In a small system, Region BVF is dominated by either Region BV or Region BF, since the optimal solution lies on the boundary between regions (see Proposition EC.2). When the in-person channel provides greater benefit, the provider should dedicate all capacity to in-person care, thereby attracting the maximum number of patients to that channel. Conversely, if the virtual channel offers higher payments, the provider should allocate most capacity to virtual services while maintaining sufficient in-person capacity to accommodate follow-up visits from virtual patients. In essence, the *pooling effect* drives the optimal choice in a small system: the provider should concentrate her limited capacity on serving a single type of patient.

In a medium-sized system, Region BVF becomes suboptimal. When the in-person payment rate r_f is sufficiently high, directing all patients to the in-person channel maximizes revenue. Conversely, when the virtual payment rate r_v is high enough, prioritizing the virtual channel—even if some patients balk—yields higher returns. When the payment gap between the two channels is moderate, however, engaging both channels to retain all patients is preferable. In this regime, the provider has enough capacity to prevent balking but must decide whether to attract additional patients who might otherwise balk, and if so, how to balance capacity between the two channels.

In a large system, no patients balk regardless of how capacity is allocated. The provider therefore focuses on steering patients toward the more profitable channel. In this case, the key decision factor is the payment or revenue margin per patient.

Theorem 2 shows that the provider's optimal capacity strategy depends on both the capacity–demand relationship within her practice and the external payment environment. In practice, few providers operate as truly large systems because healthcare resources are often limited. Most providers likely function as medium-sized systems, where balancing virtual and in-person capacity is essential for maintaining efficiency, particularly when payment rates for the two channels are relatively similar.

5 Incorporating Transportation Subsidy

In this section we explore two scenarios: one where the subsidy is paid by the provider and another in which it is covered by the government. In each scenario, we investigate the impact of transportation subsidies on the overall access rate, i.e., the proportion of patients who do not balk. This total access rate can be regarded as an indicator of social welfare (Çakıcı and Mills 2024).

5.1 Provider-Funded Transportation Subsidy

In this subsection, we study the provider's joint decision regarding the transportation subsidy b and the capacity allocation (μ_f, μ_v). Generally, we allow b to exceed the travel cost T. However, when the subsidy is paid by the provider, we impose that it does not exceed the received payment, i.e.,

$$b \le r_f \quad \text{and} \quad \delta b \le r_v. \tag{6}$$

These conditions guarantee a positive expected net revenue from each visit for the provider.

The provider seeks to solve the following optimization problem:

$$\begin{aligned} \max_{\mu_f,\mu_v,b\geq 0} \quad & (r_f - b)\lambda_f + (r_v - \delta b)\lambda_v \\ s.t. \quad & \mu_f + \mu_v = \mu, \\ & (\lambda_f, \lambda_v) \text{ is defined in Theorem EC.1.} \end{aligned} \tag{P1}$$

Problem (P1) can be reformulated as a two-stage optimization problem. In the second stage, the optimal capacity allocation is determined for a given b, and in the first stage, the optimal value of b is chosen. The second-stage problem has a structure similar to that of (P). Accordingly, the capacity constraint $\mu_f + \mu_v = \mu$ can be relaxed to $\mu_f + \mu_v \leq \mu$, as established in Lemma 1. Moreover, for any fixed b, the optimal capacity allocation follows the framework presented in Theorem 2. The main source of complexity in (P1) is that different values of b yield distinct feasible equilibrium regions, leading to different patterns of optimal capacity allocation.

We start our analysis by examining the optimal capacity allocation for an arbitrary value of b. Since the patient equilibrium exhibits a similar structure to that described in Section 3.2, all the optimization outcomes can be directly derived from the prior analysis in Section 4. Given a fixed subsidy level b, the patient equilibrium that emerges under the provider's optimal capacity allocation can take several forms depending on the interplay between system capacity, subsidy, and waiting-time sensitivities. To streamline the analysis, in Corollary EC.1 of the E-companion, we consolidate all these potential forms into four scenarios, which represent aggregated outcomes of the seven equilibrium regions characterized in Theorem EC.1. These four scenarios are: Scenario B where all patients balk, Scenario NV where no patients choose the virtual channel, Scenario NF where no patients choose the face-to-face channel, and Scenario VF where no patients balk.

Next, we identify the optimal subsidy level b within each scenario and then derive the global optimal joint decision (b^*, μ_f^*, μ_v^*), by comparing the four scenarios. For Scenario B, the analysis is straightforward because no transportation subsidy is needed, and thus the optimal b is zero. The analyses of the other three scenarios are more involved because the value of b determines which scenarios are feasible. To facilitate this analysis, we first establish the range of b that renders each scenario possible.

We begin by deriving a common upper bound $\bar{b}$ for b^*, the optimal subsidy. As detailed in Lemma EC.1 of the E-companion, excessively large subsidies make all patients willing to participate under the provider's optimal capacity allocation. The lemma provides an analytical expression for the upper bound $\bar{b}$, beyond which the system effectively operates as a "large" system (see Definition 1). In practice, this means that once b reaches $\bar{b}$, further increases yield no additional benefit in terms of patient access or provider revenue.

We subsequently refine this upper bound for b^* within each specific scenario. Let $\bar{b}_{NV}$, $\bar{b}_{NF}$, and $\bar{b}_{VF}$ denote the refined, scenario-specific upper bounds of b^* when examining the optimal subsidy under Scenario NV, NF, and VF, respectively. Each refined upper bound is defined as the minimum of three valid upper bounds. The first is the global upper bound $\bar{b}$ derived in Lemma EC.1. The second is $\min\{r_f, \delta^{-1} r_v\}$, which ensures that inequality (6) holds so that the received payment exceeds the subsidy. Because a larger subsidy b increases the total arrival rate in each scenario, but the maximum total arrival rate is Λ, this maximum defines the third valid upper bound for b. Detailed formulations for $\bar{b}_{NV}$, $\bar{b}_{NF}$, and $\bar{b}_{VF}$ are provided in E-Companion EC.1.3.1.

We next determine the lower bounds of b^* for each scenario. Let $\underline{b}_{NV}$, $\underline{b}_{NF}$, and $\underline{b}_{VF}$ represent the smallest b that makes Scenario NV, NF, and VF feasible, respectively. For instance, $\underline{b}_{NV} = \theta_f \mu^{-1} - R + T$ ensures that λ_f^{NV} remains nonnegative. The specific formulations for $\underline{b}_{NV}$, $\underline{b}_{NF}$, and $\underline{b}_{VF}$ are also provided in E-Companion EC.1.3.1.

Having characterized these bounds, we are ready to analyze b^* within each scenario described in Corollary EC.1. For Scenario B, the optimal subsidy is $b^{B*} = 0$. For each Scenario $X \in \{\text{NV}, \text{NF}, \text{VF}\}$, the provider solves

$$\max_{b \in [\underline{b}_X, \bar{b}_X]} (r_f - b)\lambda_f^X + (r_v - \delta b)\lambda_v^X, \tag{7}$$

where λ_f^X and λ_v^X are defined in Corollary EC.1, and $\underline{b}_X$ and $\bar{b}_X$ are defined in E-Companion EC.1.3.1. We use a superscript (e.g., B in b^{B*}) to denote the optimal b under each scenario.

We now examine the optimal subsidy b^{X*} corresponding to each scenario. Proposition EC.5 of the E-companion characterizes the optimal subsidy in Scenario NV, where only in-person services are offered. In this case, the provider balances the trade-off between the subsidy cost and the additional demand it generates, and the resulting closed-form solution shows that the optimal subsidy decreases with system capacity μ and increases with patient waiting-time sensitivity θ_f.

Deriving b^{NF*} for Scenario NF, where only virtual visits are offered, is more involved because of the coupled patient equilibrium between virtual and follow-up in-person visits. By imposing a mild technical condition, we obtain an explicit expression for b^{NF*}, as detailed in Proposition EC.6 of the E-companion. Essentially, this technical condition restricts the follow-up probability δ from being unrealistically high, consistent with empirically observed return rates of 6–20% (Yamamoto 2014, Uscher-Pines et al. 2016, Shi et al. 2018). The result reveals that the optimal subsidy in Scenario NF increases with the virtual channel payment r_v.

Finally, Proposition EC.7 provides the optimal subsidy for Scenario VF, in which both channels are active and no patients balk. The optimal subsidy b^{VF*} depends on the payment differential $(r_v - r_f)$ and the total service capacity μ. When the virtual payment advantage is modest, the subsidy is set to its interior optimum $\tilde{b}^{VF}$; otherwise, it reaches one of the boundary values $\underline{b}_{VF}$ or $\bar{b}_{VF}$, whichever yields higher earnings.

After obtaining the optimal subsidy within each scenario, we are able to identify the scenario that yields the highest net revenue—that is, the globally optimal subsidy and corresponding capacity allocation. To streamline this analysis, we define several capacity thresholds that determine which scenarios are feasible. Let $\underline{\mu}_X$ denote the minimum capacity required for Scenario X to occur, where $X \in \{\text{NV}, \text{NF}, \text{VF}\}$. In addition, let $\bar{\mu}$ represent the capacity threshold beyond which a subsidy is no longer necessary. Specifically, when $\mu \geq \bar{\mu}$, the provider can reach Regions F and V (see Theorem EC.1) without any subsidy. Detailed formulations of these capacity thresholds are provided in E-Companion EC.1.3.3. Using these thresholds, we can identify the optimal scenario based on the total capacity μ. Proposition 1 summarizes the results.

PROPOSITION 1 **(Optimal Scenario and Subsidy as a Function of System Size)**. *The optimal scenario and corresponding subsidy for a given total capacity μ are as follows.*

1. *When $\mu \leq \min\{\underline{\mu}_{NV}, \underline{\mu}_{NF}\}$, Scenario B is optimal, and $b^* = 0$.*
2. *When $\min\{\underline{\mu}_{NV}, \underline{\mu}_{NF}\} < \mu < \max\{\underline{\mu}_{NV}, \underline{\mu}_{NF}\}$:*
   - *If $\underline{\mu}_{NV} < \underline{\mu}_{NF}$, Scenario NF is optimal with $b^* = b^{NF*}$.*
   - *If $\underline{\mu}_{NV} > \underline{\mu}_{NF}$, Scenario NV is optimal with $b^* = b^{NV*}$.*
3. *When $\max\{\underline{\mu}_{NV}, \underline{\mu}_{NF}\} \leq \mu < \underline{\mu}_{VF}$, either Scenario NV or NF is optimal.*
4. *When $\underline{\mu}_{VF} \leq \mu < \bar{\mu}$, any of Scenarios NV, NF, or VF may be optimal.*
5. *When $\mu \geq \bar{\mu}$, no subsidy is needed ($b^* = 0$).*

Proposition 1 indicates that transportation subsidies benefit the provider only when the system size is neither too small nor too large, specifically for μ satisfying $\min\{\underline{\mu}_{NV}, \underline{\mu}_{NF}\} < \mu < \bar{\mu}$ (Cases 2–4).

To relate the case with subsidies to our baseline results without subsidies, we note that transportation subsidies reshape the provider's optimal capacity allocation, particularly in medium-sized systems. First, the subsidy b exerts heterogeneous impacts on the net reimbursement rate across service channels: the net reimbursement rate for in-person visits decreases by b, whereas that for virtual visits decreases only by δb. Second, when $b > 0$, the equilibrium boundaries defined in Section 3 shift leftward (see Figure 2), expanding the regions in which in-person service dominates. Consequently, the provider's optimal capacity allocation can differ substantially from the no-subsidy setting. This change in capacity allocation directly drives the differences in patient access rates analyzed below. Hence, the comparison of access rates can be interpreted as the aggregate effect of subsidy-induced reallocation of capacity between the two service channels.

5.1.1 Impact on Access to Care

Offering transportation subsidies helps the provider increase her revenues and improves the accessibility of the in-person channel for patients. However, its impact on *overall* patient access to care is unclear. In our analysis, we operationalize the overall patient access to care by the total effective arrival rate of patients who access the healthcare service without balking. i.e, $\lambda_f^* + \lambda_v^*$, where λ_f^* and λ_v^* are the effective arrival rates in equilibrium under the provider's optimal joint decision b^* and (μ_f^*, μ_v^*). As discussed in Çakıcı and Mills

(2024), the overall patient access to care can be viewed as a measure of social welfare. If such a subsidy, despite increasing the provider's revenue, reduces the overall patient access, its adoption deserves caution. Section 6 will investigate this issue using model parameters informed by real-world data. Here, we first derive sufficient conditions to ensure that the total access rate does *not* decline when utilizing a subsidy.

PROPOSITION 2 **(Conditions for Provider-Funded Transportation Subsidies Not Reducing Access)**. *Consider a provider that offers a transportation subsidy and jointly optimizes it with capacity allocation. For fixed model parameters, the total access rate will not decrease under any of the following conditions:*

1. ***Large system,*** *i.e., when system capacity is sufficiently high.*
2. ***Medium-sized system or small system with*** $\mu \geq \Lambda + \theta_f[R-T]^{-1}$*, if*

$$\frac{r_v}{r_f} \leq 1 \;\; or \;\; \frac{r_v}{r_f} \geq (1+\delta)\max\left\{1, \frac{\Lambda}{\mu - \frac{\theta_f}{R-T} - \frac{\theta_v}{(1-\delta)R}}\right\}. \tag{8}$$

3. ***Small system with*** $\mu < \Lambda + \theta_f[R-T]^{-1}$*, if*

$$\frac{r_v}{r_f} \leq \min\left\{1, \frac{(1+\delta)(\mu - \frac{\theta_f}{R-T})}{\mu - \frac{\left(\sqrt{\delta\theta_f}+\sqrt{\theta_v}\right)^2}{R-\delta T}}\right\} \;\; or \;\; \frac{r_v}{r_f} \geq \frac{(1+\delta)(\mu - \frac{\theta_f}{R-T})}{\mu - \frac{\theta_f}{R-T} - \frac{\theta_v}{(1-\delta)R}}.$$

In a large system, the provider has no incentive to offer a transportation subsidy, and the access rate remains unaffected. For medium-sized or small systems, when the reimbursement ratio r_v/r_f is either sufficiently small or sufficiently large, the total access rate does not decrease under the optimal subsidy.

To see this, consider the case where r_v/r_f is small. Here, the in-person channel generates higher marginal revenue, so the provider allocates all capacity to in-person care, leading to $\lambda_v^*(0) = 0$, where $\lambda_f^*(b)$ and $\lambda_v^*(b)$ denote the equilibrium arrival rates under the optimal capacity allocation for subsidy level b. Introducing the optimal subsidy allows the provider to achieve higher revenues at a lower net payment per patient, which implies an expanded total access. Specifically,

$$r_f\lambda_f^*(b^*) + r_f\lambda_v^*(b^*) \geq (r_f - b^*)\lambda_f^*(b^*) + (r_v - \delta b^*)\lambda_v^*(b^*) \geq r_f\lambda_f^*(0),$$

implying that $\lambda_f^*(b^*) + \lambda_v^*(b^*) \geq \lambda_f^*(0)$ because $r_f \geq r_v$. A symmetric argument applies when r_v/r_f is sufficiently large: the provider then favors the virtual channel, and adopting the optimal subsidy again ensures that the total access rate does not decrease.

5.2 Government-Funded Transportation Subsidy

In this subsection, we study the scenario where the subsidy is paid by the government and, therefore, incurs no cost to the provider. The provider's objective is the same as in (P), while patient equilibrium behavior is governed by Theorem EC.1 under a given subsidy b.

$$\max_{\mu_f,\, \mu_v \geq 0} \; r_f\lambda_f + r_v\lambda_v \tag{P2}$$

$$s.t.\ \ \mu_f + \mu_v = \mu,$$
$$(\lambda_f, \lambda_v) \text{ is defined in Theorem EC.1.}$$

In (P2), the provider's optimal capacity allocation can be analyzed similarly to the base model (P), with T replaced by $T - b$. All results from Theorem 2 can be readily applied.

Transportation subsidies make it easier for patients to access the in-person channel and improve patient utilities. As the provider bears no cost, one may expect transportation subsidies paid by the government would always increase patient access to care. However, the provider can capitalize on the improved patient utility and adjust her capacity allocation decisions, which affect the patient mix she serves. It is uncertain whether social welfare, measured by the total access rate, necessarily improves under government interventions.

Similar to Proposition 2, we now derive the conditions under which the total access rate will *not* decline when transportation subsidies are fully funded by the government.

PROPOSITION 3 **(Conditions for Government-Funded Transportation Subsidies Not Reducing Access).** *Suppose the government provides transportation subsidies at no cost to the provider. With all model parameters fixed, the total access rate will not decrease under any of the following conditions:*

1. ***Large system,*** *i.e., when system capacity is sufficiently high.*
2. ***Medium-sized system or small system with*** $\mu \geq \Lambda + \theta_f[R-T]^{-1}$*, if*

$$\frac{r_v}{r_f} \leq \frac{(1+\delta)\Lambda}{\min\left\{(1+\delta)\Lambda,\ \mu - \frac{\left(\sqrt{\delta\theta_f}+\sqrt{\theta_v}\right)^2}{R-\delta T+\delta b}\right\}} \quad or \quad \frac{r_v}{r_f} \geq (1+\delta)\max\left\{1,\ \frac{\Lambda}{\mu - \frac{\theta_f}{R-T} - \frac{\theta_v}{(1-\delta)R}}\right\}.$$

3. ***Small system with*** $\mu < \Lambda + \theta_f[R-T]^{-1}$*, if*

$$\frac{r_v}{r_f} \leq \min\left\{1,\ \frac{(1+\delta)(\mu - \frac{\theta_f}{R-T})}{\mu - \frac{\left(\sqrt{\delta\theta_f}+\sqrt{\theta_v}\right)^2}{R-\delta T}}\right\} \quad or \quad 1 < \frac{r_v}{r_f} \leq \frac{(1+\delta)\min\left\{\Lambda,\ \mu - \frac{\theta_f}{R-T+b}\right\}}{\mu - \frac{\left(\sqrt{\delta\theta_f}+\sqrt{\theta_v}\right)^2}{R-\delta T+\delta b}} \quad or \quad \frac{r_v}{r_f} \geq \frac{(1+\delta)(\mu - \frac{\theta_f}{R-T})}{\mu - \frac{\theta_f}{R-T} - \frac{\theta_v}{(1-\delta)R}}.$$

Intuitively, patient access to care is less likely to decline when transportation subsidies are government-funded, since the provider faces no direct cost from offering them. Indeed, the conditions in Proposition 3 are less restrictive than those in Proposition 2. Nevertheless, there is no theoretical guarantee that patient access will always improve under government-funded subsidies (more on this in Section 6.1).

6 Comparison between Provider- and Government-Funded Subsidies

In this section, we directly compare provider- and government-funded transportation subsidies in order to understand how the source of financing shapes both provider operational decisions and patient access outcomes. While both subsidy schemes reduce patients' effective travel costs and may therefore increase demand for in-person care, they generate fundamentally different incentives for the provider. Under a provider-funded subsidy, the provider internalizes the subsidy cost and adjusts capacity allocation accordingly; under a government-funded subsidy, the provider benefits from increased demand without bearing the subsidy expense. As a result, these differing incentives can induce distinct provider capacity allocations and, in turn, different implications for total patient access. We first establish analytical results that characterize how different subsidy schemes influence patient access (Proposition 4), and then conduct numerical experiments to quantify their impact on provider operations and to illustrate conditions under which subsidies improve—or paradoxically reduce—overall patient access.

PROPOSITION 4 **(Provider- vs. Government-Funded Access)**. *For any given set of model parameters, there exists a government-funded transportation subsidy under which the total patient access rate is not smaller than that achieved under any provider-funded transportation subsidy designed to maximize revenue. Nevertheless, when $\mu > (1+\delta)\Lambda$ and both r_f and r_v exceed certain thresholds, the revenue-maximizing provider-funded transportation subsidy also attains the maximum patient access rate of Λ.*

Proposition 4 indicates that, in terms of maximizing access to care, a properly designed government-funded transportation subsidy generally outperforms a provider-funded subsidy aimed at revenue maximization. Government-funded subsidies are primarily designed to compensate patients by reducing their travel costs and thus to facilitate access to care, whereas provider-funded subsidies are strategic instruments that providers can use to influence patient demand and boost revenue. When service capacity or reimbursement rates are low, higher revenue does not necessarily translate into improved care access.

However, when providers have enough capacity to accommodate all patients and reimbursement rates are sufficiently high, a revenue-maximizing provider-funded transportation subsidy can attain an access rate equivalent to that of the optimal government-funded transportation subsidy chose to maximize access. Given the potential information asymmetry between the government and healthcare providers, as well as practical policy constraints (e.g., the need to apply a uniform subsidy structure across many providers operating in heterogeneous environments), designing the optimal government subsidy can be challenging for policymakers. Consequently, letting providers design their own subsidies may lead to better access outcomes, particularly when provider capacity and reimbursement levels are high.

6.1 Numerical Study

To project the potential practical implications of our analytical results, we conduct an extensive numerical study informed by real-world data and empirical evidence. We start by describing the calibration of our model parameters. We then compare provider capacity allocations, revenues, and patient access to care across different subsidy schemes. Finally, given that the primary objective of government-funded subsidies is to support patient care, we examine their impact on patient access.

To populate our model, we obtain realistic baseline values for the following parameters: transportation cost T, waiting cost rates θ_f and θ_v, probability of requiring a supplementary in-person visit after completing a virtual visit δ, and reimbursement rates r_f and r_v.

For transportation costs, we distinguish between rural and non-rural areas. A recent research report by the Rural Health Research Gateway (Ostmo and Rosencrans 2022) finds that rural residents travel, on average, 17.8 miles for care compared to 8.1 miles for non-rural residents. Rural residents spend an average of 34.2 minutes traveling for care, compared to 25.5 minutes for non-rural residents. To convert these transportation efforts into monetary values, we use the 2023 IRS standard mileage rates to calculate travel costs and use wage to measure the value of time, following the classic economics literature (Becker 1965). Specifically, we consider a millage rate of \$0.655 per mile and use the average hourly wage of \$35 reported by the U.S. Bureau of Labor Statistics (2025). While our calibration relies on IRS mileage rates and average travel distances to estimate transportation costs, a related real-world benchmark is Medicaid's Non-Emergency Medical Transportation (NEMT) program, which reimburses beneficiaries for mileage and other travel expenses associated with accessing care. According to a CMS fact sheet, Medicaid programs are required to cover NEMT to and from medical services and may reimburse beneficiaries for mileage when they use personal vehicles (Centers for Medicare & Medicaid Services 2023). State-level guidelines also indicate that NEMT mileage reimbursement rates are often set at fixed per-mile amounts comparable to the IRS standard mileage rate. For example, Michigan's 2025 NEMT schedule reimburses at \$0.70 per mile (Michigan Department of Health and Human Services 2025). Although detailed reimbursement data are not readily available for direct parameterization, our use of the IRS mileage rate ensures that the magnitude of the modeled subsidy is aligned with existing travel-support programs such as NEMT. Consequently, we set the transportation costs, respectively, for rural and non-rural areas as:

$$T_{\text{rural}} = 17.8 \cdot 0.655 + \frac{34.2 \cdot 35}{60} \approx \$32 \quad \text{and} \quad T_{\text{non-rural}} = 8.1 \cdot 0.655 + \frac{25.5 \cdot 35}{60} \approx \$20.$$

We set the in-person waiting cost per hour to be $\theta_f = 35$, matching the hourly wage value. In a call center study, Hathaway et al. (2021) find that callers experience three to six times less discomfort per unit of time while waiting for callbacks than while waiting in a queue. This suggests that virtual waiting (akin to waiting for callbacks) is less costly than in-person waiting (similar to waiting in a queue). Following this rationale, we set $\theta_v = 12$, about one-third of θ_f.

The probability of requiring a supplementary in-person visit is set as $\delta = 5\%$, which is consistent, though conservative, with previous empirical findings (Gordon et al. 2017, Ashwood et al. 2017, Shi et al. 2018, Li et al. 2021). Furthermore, we use the out-of-pocket cost for an outpatient care visit as a proxy for the utility of seeing the provider. Such costs range from \$32 to \$175 per visit (Rohatsch 2025). In our numerical analysis, we use a moderate value of $R = \$60$.

To set the reimbursement rates, we consider the most-billed office visit CPT code 99214, reimbursed at about \$100 per visit (Kim 2023). Accordingly, we set $r_f = \$100$. Telehealth reimbursements are typically equal or lower than in-person rates. To be concrete, we consider that virtual visits are reimbursed at 80% of the in-person rate, following the reimbursement policy of Point32Health (Point32Health, Inc. 2024), one of the top twenty health plans in the U.S. Given $\delta = 5\%$, the expected reimbursement for a virtual visit is $r_v = 80 + 100 \times 5\% = \$85$. Lastly, we consider a practice with an average of 20 patient arrivals per 8-hour day, i.e., the hourly arrival rate is set as $\Lambda = 2.5$.

Table 2 reports the relative access rate (i.e., the proportion of patients who receive services, defined as $(\lambda_f^* + \lambda_v^*)/\Lambda$), the optimal capacity allocation (μ_f^*, μ_v^*), total revenue, and the corresponding subsidy levels for representative values of δ and r_v/r_f around their baselines, under two system sizes ($\mu = 2.75$ and $\mu = 3.3$). Results are presented for three regimes: the no-subsidy baseline, the optimal provider-funded subsidy, and the optimal government-funded subsidy.

The optimal provider-funded subsidy (b^*_{pro}) is chosen by the provider to maximize revenue, following the characterization in Section 5.1. In contrast, the optimal government-funded subsidy (b^*_{gov}) is selected to maximize total access; this subsidy corresponds to the minimum subsidy required to attain the highest achievable access level, given that capacity allocation is determined by the provider's decision problem described in Section 5.2. By construction, the relative access rate under the optimal government-funded regime is guaranteed to be no lower than that in the no-subsidy case. In contrast, the results confirm the possibility of a backfire effect under the optimal provider-funded subsidies: when the subsidy is chosen by the provider to maximize revenue, total access can fall below the no-subsidy benchmark.

Next, we discuss the access rate under the two subsidy regimes. When $\mu = 2.75$, the provider-funded subsidy shifts capacity toward in-person visits and may reduce access when r_v/r_f is low (i.e., when reimbursement for virtual visits is relatively low compared to in-person visits). In contrast, the government-funded subsidy improves access without lowering revenue. When $\mu = 3.3$, access rates are close to one under both regimes, indicating that subsidies have limited impact once capacity becomes abundant.

Table 2 also reports the optimal subsidy levels. When access is not maximized under the no-subsidy regime, the government—whose objective is to maximize access—provides a high subsidy to raise the access rate under the government-funded regime. In contrast, under the provider-funded regime, the provider may either set a positive subsidy or choose not to offer one, since her objective is revenue maximization rather than access expansion. If she does offer a positive subsidy, the resulting access rate may be either higher or lower than under the no-subsidy regime.

When access is already maximized under the no-subsidy regime, the government will not provide a subsidy, as the access rate is already at its highest attainable level. However, under the provider-funded regime, the provider may still choose to offer a positive subsidy to increase revenue, even if this reduces

Table 2 Comparison of Subsidy Regimes

			No subsidy			Provider-funded				Government-funded			
μ	δ	r_v/r_f	**Acc. Rate**	(μ_f^*, μ_v^*)	**Rev.**	**Acc. Rate**	(μ_f^*, μ_v^*)	**Rev.**	b^*_{pro}	**Acc. Rate**	(μ_f^*, μ_v^*)	**Rev.**	b^*_{gov}
2.75	3%	0.70	0.94	(0.15, 2.60)	163.70	0.75	(2.75, 0.00)	165.01	12.36	0.94	(2.75, 0.00)	235.23	60.00
		0.85	0.94	(0.15, 2.60)	198.78	0.94	(0.15, 2.60)	198.78	0.00	0.94	(2.75, 0.00)	235.23	60.00
		0.95	0.94	(0.15, 2.60)	222.16	0.94	(0.15, 2.60)	222.16	0.00	0.94	(2.75, 0.00)	235.23	60.00
	5%	0.70	0.90	(0.22, 2.53)	157.17	0.75	(2.75, 0.00)	165.01	12.36	0.94	(2.75, 0.00)	235.23	60.00
		0.85	0.90	(0.22, 2.53)	190.85	0.90	(0.22, 2.53)	190.85	0.00	0.94	(2.75, 0.00)	235.23	60.00
		0.95	0.90	(0.22, 2.53)	213.31	0.90	(0.22, 2.53)	213.31	0.00	0.94	(2.75, 0.00)	235.23	60.00
	7%	0.70	0.86	(0.29, 2.46)	151.26	0.75	(2.75, 0.00)	165.01	12.36	0.94	(2.75, 0.00)	235.23	60.00
		0.85	0.86	(0.29, 2.46)	183.67	0.86	(0.29, 2.46)	183.67	0.00	0.94	(2.75, 0.00)	235.23	60.00
		0.95	0.86	(0.29, 2.46)	205.28	0.86	(0.29, 2.46)	205.28	0.00	0.94	(2.75, 0.00)	235.23	60.00
3.30	3%	0.70	0.82	(3.30, 0.00)	205.00	0.94	(3.30, 0.00)	214.22	8.85	1.00	(3.30, 0.00)	250.00	16.00
		0.85	1.00	(0.16, 3.14)	212.50	0.94	(3.30, 0.00)	214.22	8.85	1.00	(0.16, 3.14)	212.50	0.00
		0.95	1.00	(0.16, 3.14)	237.50	1.00	(0.16, 3.14)	237.50	0.00	1.00	(0.16, 3.14)	237.50	0.00
	5%	0.70	0.82	(3.30, 0.00)	205.00	0.94	(3.30, 0.00)	214.22	8.85	1.00	(3.30, 0.00)	250.00	16.00
		0.85	1.00	(0.25, 3.05)	212.50	0.94	(3.30, 0.00)	214.22	8.85	1.00	(0.25, 3.05)	212.50	0.00
		0.95	1.00	(0.25, 3.05)	237.50	1.00	(0.25, 3.05)	237.50	0.00	1.00	(0.25, 3.05)	237.50	0.00
	7%	0.70	0.82	(3.30, 0.00)	205.00	0.94	(3.30, 0.00)	214.22	8.85	1.00	(3.30, 0.00)	250.00	16.00
		0.85	1.00	(0.32, 2.98)	212.50	0.94	(3.30, 0.00)	214.22	8.85	1.00	(0.32, 2.98)	212.50	0.00
		0.95	1.00	(0.32, 2.98)	237.50	1.00	(0.32, 2.98)	237.50	0.00	1.00	(0.32, 2.98)	237.50	0.00

Note. $\Lambda = 2.5$, $T = 32$, $R = 60$, $\theta_f = 35$, $\theta_v = 12$, $r_f = 100$, and government-funded subsidies are optimized over $b_{\text{gov}} \in [0, 60]$, a sufficiently large range that allows $b > T$. Acc. Rate is the proportion of patients who receive services, i.e., $(\lambda_f^* + \lambda_v^*)/\Lambda$; (μ_f^*, μ_v^*) represents the optimal capacity allocation; Rev. stands for net revenue; b^*_{pro} and b^*_{gov} denote the optimal government- and provider-funded subsidies, respectively.

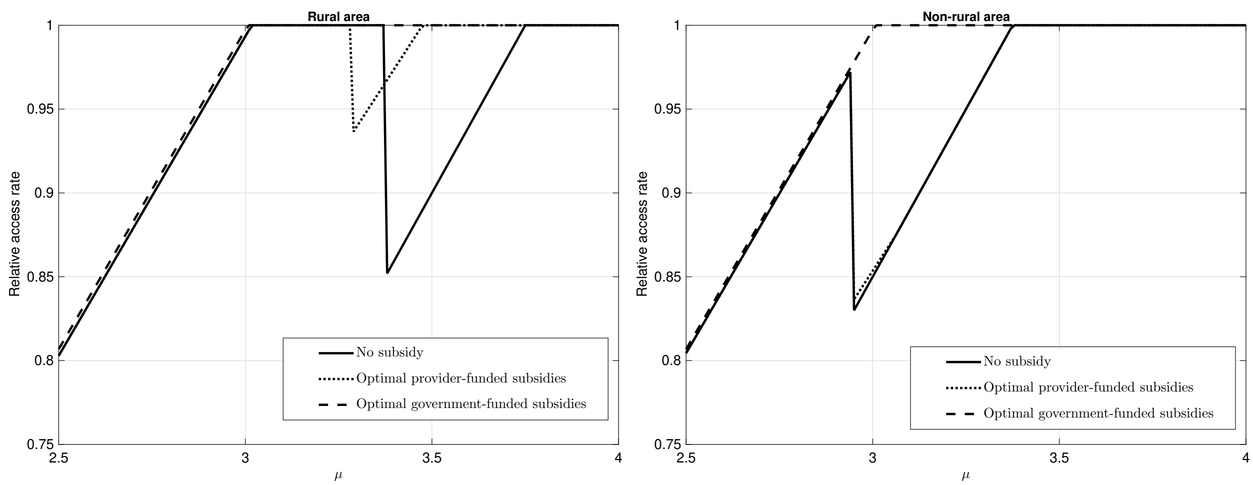
access. This difference again reflects the distinct objectives of the two regimes: the government seeks to maximize access, whereas the provider aims to maximize revenue.

Finally, in terms of revenue, the provider-funded subsidy always yields revenue at least as high as in the no-subsidy regime, since it is chosen to maximize the provider's earnings for the given capacity level. Nevertheless, revenue under the government-funded subsidy need not be lower. In our calibration, when $\mu = 2.75$, the access-maximizing government-funded subsidy increases access at no cost to the provider and therefore generates even higher revenue than the provider-funded subsidy. When $\mu = 3.30$, where capacity is relatively abundant, the provider-funded subsidy often yields the highest revenue because it enables the provider to induce a more profitable mix of patient visits.

Taken together, these results highlight that the impact of subsidies on patient access to care is driven by the provider's capacity and its allocation incentives. When capacity is relatively tight, subsidies can meaningfully alter the provider's preferred channel mix, leading to either improved access (under government funding) or reduced access (under provider funding). As capacity becomes abundant, however, access approaches its maximum level regardless of the subsidy regime, and the function of subsidies shifts primarily toward revenue reshaping rather than access expansion.

To further illustrate the role of the provider's capacity, we create a more granular figure to show how the relative access rate varies with μ under different subsidy regimes. In Figure 4, the solid line represents the no-subsidy benchmark, the dotted line corresponds to the provider-funded subsidy optimized for revenue maximization, and the dashed line shows the highest attainable access rate under the government-funded subsidy regime. To construct the dashed line, we first determine the provider's optimal capacity allocation for each possible value of b (as described in Section 5.3), and then identify the subsidy level that yields the highest access rate.

Figure 4 Comparison of access rates with and without transportation subsidies for rural and non-rural areas.

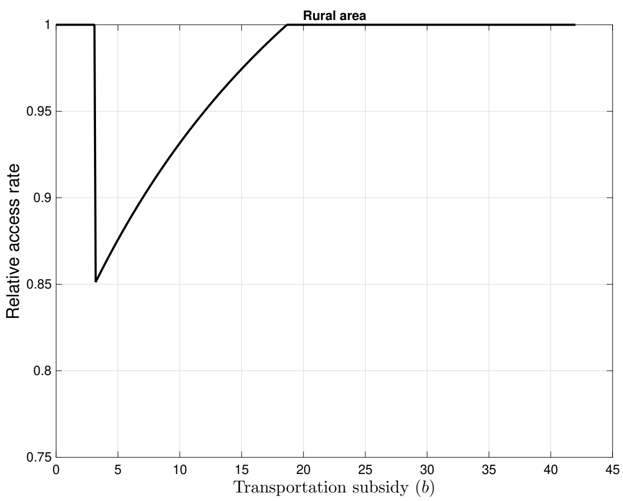


In the rural area, the dotted curve (provider-funded subsidy) occasionally falls below the solid curve, echoing our earlier findings that patient access may decline when the provider offers transportation support to maximize her revenue. This effect is most pronounced when $\mu \approx 3.2$–3.3, a range in which transportation subsidies make the in-person channel particularly attractive because transportation costs are high. By offering subsidies, the provider can generate higher revenue with fewer total patient visits through the in-person channel—which, in turn, reduces overall patient access. For comparison, the dashed curve shows the access rate under a government-funded subsidy chosen to maximize access. Together with Table 2, Figure 4 underscores that the welfare implications of transportation subsidies depend critically on whose objective governs the subsidy decision: when the subsidy is designed to facilitate in-person visits, it helps improve patient access to care; when it is used as a revenue-maximizing instrument, it may distort capacity allocation in ways that reduce overall access.

Lastly, we examine the impact of government-funded subsidies on patient access. Figure 5 illustrates how the relative access rate changes at different levels of transportation subsidies offered by the government. The total service capacity is fixed at $\mu = 3.25$. One striking observation emerges in the left panel of Figure 5:

in rural areas, patient access to care may even decline with government-funded transportation subsidy at *no* cost to the provider! This arises from the changes in the provider's optimal operational strategy. When travel costs are high and no subsidies are available, it is challenging to draw sufficient patients to the in-person channel with a higher reimbursement rate. As a result, the provider primarily operates the virtual channel with a lower reimbursement rate to serve more patients. Conversely, with subsidies, it becomes easier to attract patients to the more "profitable" channel. Even though fewer patients are served, the overall revenue could still increase. Hence, the provider adjusts her operational strategy, which may lead to a decrease in the overall access.

Figure 5 Comparison of access rates with government-funded subsidies for rural and non-rural areas.



7 Conclusion

The use of telemedicine skyrocketed during the COVID-19 pandemic, offering convenience and reduced travel costs. However, it may not fully address patient needs, sometimes necessitating supplementary in-person visits. As the pandemic has subsided, in-person care has regained favor. Transportation support, funded by providers or the government, has emerged as a key enabler of in-person access. Meanwhile, policy discussions are shifting from pay parity, originally designed to promote telemedicine adoption, to pay equity, which aims to align reimbursement with provider effort and patient outcomes. In this changing landscape for telemedicine practice, we study how an outpatient care provider can optimally balance virtual and in-person services and determine whether and how to engage with transportation subsidies. To answer these research questions, we develop a stylized queueing-game model to capture the operations of a single revenue-maximizing provider serving patients who strategically choose between service channels.

Managerial and Policy Insights. Our analysis reveals that optimal system design depends critically on provider size. Small and large systems are most efficient when focusing on a single channel, while medium-sized systems benefit from maintaining both virtual and in-person services together with transportation support for patients. Transportation subsidies are, therefore, most relevant for these medium-sized systems.

Our study highlights the impact of transportation support on patient access to care. Interestingly, transportation support, whether funded by the provider or by the government, does *not* always improve access to care. Provider-funded subsidies can increase revenue but may inadvertently reduce total access, as they may drive providers to shift capacity toward the higher-reimbursement in-person channel and serve fewer patients overall. Similarly, poorly designed government-funded subsidies may also hurt access to care in settings with high travel costs. Together, these results underscore that the benefits of transportation support depend on the interaction between provider payment incentives and patient demand behavior.

When the reimbursement gap between in-person and virtual visits is sufficiently large, the increased patient utility from transportation support helps the provider to attract more visits, ensuring that both revenue and access rise together. This finding aligns with the principle of *payment equity* to set reimbursement rates that reflect service intensity rather than parity between modalities (Shachar et al. 2020). In the post-pandemic era, properly designed payment and subsidy policies can help expand overall access to care as providers reinvest in in-person care.

Future Research Directions. Several promising avenues remain for future work. One direction is to examine additional operational levers, such as patient prioritization or triage mechanisms. Another is to consider systems in which returning patients are served through a separate channel with dedicated capacity. Finally, the design of reimbursement policies that better align provider incentives with social welfare objectives represents a fruitful direction for continued research.

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E-Companion for Channel Management in Outpatient Care: Implications of Telemedicine and Transportation Support

EC.1 Additional Technical Results

EC.1.1 Additional Technical Results for Section 3

DEFINITION EC.1.

- Region B:

$$\frac{\theta_v}{\mu_v} + \frac{\delta\theta_f}{\mu_f} \geq R - \delta T + \delta b, \tag{EC.1}$$

$$\mu_f \leq \frac{\theta_f}{R - T + b}. \tag{EC.2}$$

- Region V:

$$\frac{\theta_v}{\mu_v - \Lambda} - \frac{(1-\delta)\theta_f}{\mu_f - \delta\Lambda} \leq (1-\delta)(T-b) \quad \text{for } \mu_f > \delta\Lambda, \tag{EC.3}$$

$$\frac{\theta_v}{\mu_v - \Lambda} + \frac{\delta\theta_f}{\mu_f - \delta\Lambda} \leq R - \delta T + \delta b, \tag{EC.4}$$

$$\mu_f > \delta\Lambda, \tag{EC.5}$$

$$\mu_v > \Lambda. \tag{EC.6}$$

- Region F:

$$\mu_f \geq \Lambda + \frac{\theta_f}{R - T + b}, \tag{EC.7}$$

$$\frac{\theta_v}{\mu_v} - \frac{(1-\delta)\theta_f}{\mu_f - \Lambda} \geq (1-\delta)(T-b). \tag{EC.8}$$

- Region BVF:

$$\mu_v \geq \frac{\theta_v}{(1-\delta)R}, \tag{EC.9}$$

$$\mu_f - \delta\mu_v \geq \frac{\theta_f}{R - T + b} - \frac{\delta\theta_v}{(1-\delta)R}, \tag{EC.10}$$

$$\mu_f + (1-\delta)\mu_v \leq \Lambda + \frac{\theta_f}{R - T + b} + \frac{\theta_v}{R}. \tag{EC.11}$$

- Region BF:

$$\mu_f \geq \frac{\theta_f}{R - T + b}, \tag{EC.12}$$

$$\mu_f \leq \Lambda + \frac{\theta_f}{R - T + b}, \tag{EC.13}$$

$$\mu_v \leq \frac{\theta_v}{(1-\delta)R}. \tag{EC.14}$$

- Region BV:

$$\frac{\theta_v}{\mu_v} + \frac{\delta\theta_f}{\mu_f} \leq R - \delta T + \delta b, \tag{EC.15}$$

$$\frac{\theta_v}{\mu_v - \Lambda} + \frac{\delta\theta_f}{\mu_f - \delta\Lambda} \geq R - \delta T + \delta b \text{ for } \mu_v > \Lambda \text{ and } \mu_f > \delta\Lambda, \tag{EC.16}$$

$$\mu_f - \delta\mu_v \leq \frac{\theta_f}{R - T + b} - \frac{\delta\theta_v}{(1-\delta)R}. \tag{EC.17}$$

- Region VF:

$$\frac{\theta_v}{\mu_v} - \frac{(1-\delta)\theta_f}{\mu_f - \Lambda} \leq (1-\delta)(T-b) \quad \text{for } \mu_f > \Lambda, \tag{EC.18}$$

$$\frac{\theta_v}{\mu_v - \Lambda} - \frac{(1-\delta)\theta_f}{\mu_f - \delta\Lambda} \geq (1-\delta)(T-b) \quad \text{for } \mu_v > \Lambda, \tag{EC.19}$$

$$\mu_f + (1-\delta)\mu_v \geq \Lambda + \frac{\theta_f}{R - T + b} + \frac{\theta_v}{R}, \tag{EC.20}$$

$$\mu_f > \delta\Lambda. \tag{EC.21}$$

THEOREM EC.1 **(Equilibrium Characterization with a Fixed Subsidy** b**).** *The seven regions of* (μ_f, μ_v) *defined in Definition EC.1 are mutually exclusive and collectively exhaustive. And in each region, there exists a unique equilibrium strategy such that*

1. *When* $(\mu_f, \mu_v) \in$ *Region B, there exists a unique equilibrium strategy – strategy B – such that* $\lambda_f = \lambda_v = 0$.
2. *When* $(\mu_f, \mu_v) \in$ *Region V, there exists a unique equilibrium strategy – strategy V – such that* $\lambda_f = 0$ *and* $\lambda_v = \Lambda$.
3. *When* $(\mu_f, \mu_v) \in$ *Region F, there exists a unique equilibrium strategy – strategy F – such that* $\lambda_f = \Lambda$ *and* $\lambda_v = 0$.
4. *When* $(\mu_f, \mu_v) \in$ *Region BVF, there exists a unique equilibrium strategy – strategy BVF – such that*

$$\lambda_f = \mu_f - \delta\mu_v - \frac{\theta_f}{R - T + b} + \frac{\delta\theta_v}{(1-\delta)R}$$

and

$$\lambda_v = \mu_v - \frac{\theta_v}{(1-\delta)R}.$$

5. *When* $(\mu_f, \mu_v) \in$ *Region BF, there exists a unique equilibrium strategy – strategy BF – such that*

$$\lambda_f = \mu_f - \frac{\theta_f}{R - T + b}$$

and $\lambda_v = 0$.

6. *When* $(\mu_f, \mu_v) \in$ *Region BV, there exists a unique equilibrium strategy – strategy BV – such that* $\lambda_f = 0$ *and*

$$\lambda_v = \frac{\mathcal{B} - \sqrt{\mathcal{D}^2 + 4\delta^2\theta_f\theta_v}}{2\mathcal{A}},$$

where

$$\mathcal{A} = (R - \delta T + \delta b)\delta,$$

$$\mathcal{B} = (R - \delta T + \delta b)(\mu_f + \delta\mu_v) - \delta(\theta_f + \theta_v),$$

and

$$\mathcal{D} = (R - \delta T + \delta b)(\mu_f - \delta\mu_v) - \delta(\theta_f - \theta_v).$$

7. *When* $(\mu_f, \mu_v) \in$ *Region VF, there exists a unique equilibrium strategy – strategy VF – such that*

$$\lambda_f = \Lambda - \frac{-\tilde{\mathcal{B}} + \sqrt{\tilde{\mathcal{D}}^2 + 4\theta_f\theta_v}}{2\tilde{\mathcal{A}}}$$

and

$$\lambda_v = \frac{-\tilde{\mathcal{B}} + \sqrt{\tilde{\mathcal{D}}^2 + 4\theta_f\theta_v}}{2\tilde{\mathcal{A}}},$$

where

$$\tilde{\mathcal{A}} = (1 - \delta)(T - b),$$

$$\tilde{\mathcal{B}} = (\theta_f + \theta_v) + (T - b)(\mu_f - \Lambda - (1 - \delta)\mu_v),$$

and

$$\tilde{\mathcal{D}} = (\theta_f - \theta_v) + (T - b)(\mu_f - \Lambda + (1 - \delta)\mu_v).$$

EC.1.2 Additional Technical Results for Section 4

PROPOSITION EC.1 **(Optimal Capacity Allocation in Regions B, V, F and BF)**. *In Regions B, V and F, any feasible allocation is optimal. In addition,* $(\lambda_f^{B*}, \lambda_v^{B*}) = (0, 0)$, $(\lambda_f^{V*}, \lambda_v^{V*}) = (0, \Lambda)$ *and* $(\lambda_f^{F*}, \lambda_v^{F*}) = (\Lambda, 0)$. *For Region BF,* $(\mu_f^{BF*}, \mu_v^{BF*}) = (\bar{\mu}_f^{BF}, \mu - \bar{\mu}_f^{BF})$ *where* $\bar{\mu}_f^{BF}$ *is the largest* μ_f *such that* $(\mu_f, \mu - \mu_f) \in$ *Region BF, and*

$$(\lambda_f^{BF*}, \lambda_v^{BF*}) = (\bar{\mu}_f^{BF} - \frac{\theta_f}{R - T}, 0).$$

In Region BVF, the equilibrium arrival rates increase linearly with capacity, suggesting a bang–bang allocation in which all capacity is assigned to the more profitable channel.

Proposition EC.2 **(Optimal Capacity Allocation subject to Region BVF)**. *Let* $[\underline{\mu}_v^{BVF}, \bar{\mu}_v^{BVF}]$ *denote the range for* μ_v *such that* $(\mu - \mu_v, \mu_v) \in$ *Region BVF.*

The optimal capacity allocation depends on the payment rates such that if $r_v \geq (1+\delta) r_f$*, then* $(\mu_f^{BVF*}, \mu_v^{BVF*}) = (\mu - \bar{\mu}_v^{BVF}, \bar{\mu}_v^{BVF})$*; otherwise,* $(\mu_f^{BVF*}, \mu_v^{BVF*}) = (\mu - \underline{\mu}_v^{BVF}, \underline{\mu}_v^{BVF})$*. The corresponding equilibrium arrival rates*

$$(\lambda_f^{BVF*}, \lambda_v^{BVF*}) = (\mu_f^{BVF*} - \frac{\theta_f}{R-T} - \delta \mu_v^{BVF*} + \frac{\delta\theta_v}{(1-\delta)R}, \mu_v^{BVF*} - \frac{\theta_v}{(1-\delta)R}).$$

Based on the equilibrium outcomes in Theorem EC.1, deriving the optimal capacity allocation in Region BV is intricate, as it depends on whether Region BV dominates or is dominated by adjacent Regions V and BVF.

Proposition EC.3 **(Optimal Capacity Allocation subject to Region BV)**.

Define

$$\tilde{\mu}_v^{BV} = \frac{\mu}{1+\delta} - \frac{\delta(\theta_f - \theta_v) + (1-\delta)\sqrt{\delta\theta_f\theta_v}}{(1+\delta)(R-\delta T)}.$$

Then, we have:

– *If*

$$\mu \geq \Lambda(1+\delta) + \min\{\frac{\theta_f}{R-T} + \frac{\theta_v}{(1-\delta)R}, \frac{(\sqrt{\delta\theta_f} + \sqrt{\theta_v})^2}{R-\delta T}\},$$

any feasible allocation lying on the boundary between Region BV and Region V is optimal, leading to $(\lambda_f^{BV*}, \lambda_v^{BV*}) = (0, \Lambda)$.

– *If*

$$\mu < \Lambda(1+\delta) + \min\{\frac{\theta_f}{R-T} + \frac{\theta_v}{(1-\delta)R}, \frac{(\sqrt{\delta\theta_f} + \sqrt{\theta_v})^2}{R-\delta T}\}$$

and $(\mu - \tilde{\mu}_v^{BV}, \tilde{\mu}_v^{BV}) \in$ *Region BV, then* $(\mu_f^{BV*}, \mu_v^{BV*}) = (\mu - \tilde{\mu}_v^{BV}, \tilde{\mu}_v^{BV})$ *and*

$$\lambda_v^{BV*} = \frac{\mu}{[1+\delta} - \frac{(\sqrt{\delta\theta_f} + \sqrt{\theta_v})^2}{(1+\delta)(R-\delta T)}.$$

– *Otherwise,* $(\mu_f^{BV*}, \mu_v^{BV*}) = (\mu - \underline{\mu}_v^{BV}, \underline{\mu}_v^{BV})$*, where* $\underline{\mu}_v^{BV}$ *is the lower bound for* μ_v *such that* $(\mu - \mu_v, \mu_v) \in$ *Region BV, leading to*

$$\lambda_v^{BV*} = \frac{\mu}{1+\delta} - \frac{\theta_f}{(1+\delta)(R-T)} - \frac{\theta_v}{(1+\delta)(1-\delta)R}.$$

Proposition EC.4 **(Optimal Capacity Allocation subject to Region VF)**. *Let* $[\underline{\mu}_v^{VF}, \bar{\mu}_v^{VF}]$ *denote the range for* μ_v *such that* $(\mu - \mu_v, \mu_v) \in$ *Region VF.*

– *If* $r_v > r_f$, $(\mu_f^{VF*}, \mu_v^{VF*}) = (\mu - \bar{\mu}_v^{VF}, \bar{\mu}_v^{VF})$*, then*

$$\lambda_v^{VF*} = \min\{\Lambda, \frac{\mu}{\delta} - \frac{\Lambda}{\delta} - \frac{\theta_f}{\delta(R-T)} - \frac{\theta_v}{\delta(1-\delta)R}\}$$

and $\lambda_f^{VF*} = \Lambda - \lambda_v^{VF*}$.

– *If $r_v = r_f$, any feasible allocation is optimal and $r_f\lambda_f^{VF*} + r_v\lambda_v^{VF*} = r_f\Lambda = r_v\Lambda$.*
– *If $r_v < r_f$, $(\mu_f^{VF*}, \mu_v^{VF*}) = (\mu - \underline{\mu}_v^{VF}, \underline{\mu}_v^{VF})$ and $(\lambda_f^{VF*}, \lambda_v^{VF*}) = (\Lambda, 0)$.*

The optimal capacity allocation in Region VF depends on the relative payment rates, with capacity directed toward the channel offering the higher reimbursement.

DEFINITION EC.2. We define $\underline{\mu}^{BV}$ as the minimum μ required to attain Region BV. It is obtained by the solution of μ to

$$R - \delta T - \frac{\theta_v}{\Phi(\mu)} - \frac{\delta\theta_f}{\mu - \Phi(\mu)} = 0,$$

where

$$\Phi(\mu) = \max\left\{\frac{\theta_v}{(1-\delta)R}, \frac{\mu(\sqrt{\delta\theta_v\theta_f} - \theta_v)}{\delta\theta_f - \theta_v}\right\}. \tag{EC.22}$$

EC.1.3 Additional Technical Results for Section 5

EC.1.3.1 Optimal Capacity Allocation and the Bounds for Optimal Subsidy

Corollary EC.1 builds on Theorem EC.1 by identifying which equilibrium scenarios can arise when the provider chooses her optimal capacity allocation for a given subsidy b.

COROLLARY EC.1 **(Equilibrium under the Provider's Optimal Capacity Allocation Given b)**. *Given a subsidy b, the optimal capacity allocation $(\mu_f^*(b), \mu_v^*(b))$ should lead to one of the following equilibrium scenarios.*

1. ***Scenario B**: All patients balk.*
2. ***Scenario NV**: No patient chooses the virtual channel. Therefore, $\lambda_v^{NV} = 0$ and*

$$\lambda_f^{NV} = \min\{\Lambda, \mu - \frac{\theta_f}{R - T + b}\}.$$

3. ***Scenario NF**: No patient chooses the in-person channel. Therefore, $\lambda_f^{NF} = 0$ and*

$$\lambda_v^{NF} = \begin{cases} \min\left\{\Lambda, \frac{\mu}{1+\delta} - \frac{(\sqrt{\delta\theta_f} + \sqrt{\theta_v})^2}{(1+\delta)(R - \delta T + \delta b)}\right\}, & \textit{if } \frac{\delta(\theta_f - \theta_v) + (1-\delta)\sqrt{\delta\theta_f\theta_v}}{(R - \delta T + \delta b)} \le \frac{\theta_f}{R - T + b} - \frac{\delta\theta_v}{(1-\delta)R}, \\ \min\left\{\Lambda, \frac{\mu}{1+\delta} - \frac{\theta_f}{(1+\delta)(R - T + b)} - \frac{\theta_v}{(1+\delta)(1-\delta)R}\right\}, & \textit{otherwise.} \end{cases} \tag{EC.23}$$

4. ***Scenario VF**: No patient balks. Therefore, $\lambda_f^{VF} = \Lambda - \lambda_v^{VF}$ and*

$$\lambda_v^{VF} = \frac{1}{\delta}\left(\mu - \Lambda - \frac{\theta_v}{(1-\delta)R} - \frac{\theta_f}{R - T + b}\right). \tag{EC.24}$$

LEMMA EC.1. *The optimal subsidy b^* must satisfy $b^* \le \bar{b}$ where*

$$\bar{b} = \begin{cases} \infty, & \textit{if } \mu \le (1+\delta)\Lambda + \frac{\theta_v}{(1-\delta)R}, \\ \frac{\theta_f}{\mu - (1+\delta)\Lambda - \frac{\theta_v}{(1-\delta)R}} - R + T, & \textit{otherwise.} \end{cases} \tag{EC.25}$$

Next, we derive the upper bounds for each scenario.

For Scenario NV, recall that in Corollary EC.1, we have

$$\lambda_f^{NV} = \min\{\Lambda, \mu - \frac{\theta_f}{R - T + b}\}.$$

To make sure λ_f^{NV} just to hit Λ, we have

$$b \leq \frac{\theta_f}{\mu - \Lambda} - R + T$$

when $\mu > \Lambda$. Since

$$\frac{\theta_f}{\mu - \Lambda} - R + T < \bar{b},$$

then we have

$$\bar{b}_{NV} = \begin{cases} r_f, & \text{if } \mu \leq \Lambda, \\ \min\{r_f, \frac{\theta_f}{\mu - \Lambda} - R + T\}, & \text{otherwise.} \end{cases} \tag{EC.26}$$

In Scenario NF, the formulation of λ_f^{NF} derived in Corollary EC.1, yields that

$$\lambda_v^{NF} \leq \min\left\{\Lambda, \frac{\mu}{1+\delta} - \frac{(\sqrt{\delta\theta_f} + \sqrt{\theta_v})^2}{(1+\delta)(R - \delta T + \delta b)}\right\}. \tag{EC.27}$$

When

$$b = \frac{(\sqrt{\delta\theta_f} + \sqrt{\theta_v})^2}{\delta[\mu - (1+\delta)\Lambda]} - \frac{R}{\delta} + T,$$

the RHS of (EC.27) just hits Λ if $\mu > (1+\delta)\Lambda$. Finally,

$$\bar{b}_{NF} = \begin{cases} \frac{r_v}{\delta}, & \text{if } \mu \leq (1+\delta)\Lambda, \\ \min\{\frac{r_v}{\delta}, \frac{(\sqrt{\delta\theta_f} + \sqrt{\theta_v})^2}{\delta[\mu - (1+\delta)\Lambda]} - \frac{R}{\delta} + T, & \text{if } (1+\delta)\Lambda < \mu \leq (1+\delta)\Lambda + \frac{\theta_v}{(1-\delta)R}\}, \\ \min\{\frac{r_v}{\delta}, \bar{b}, \frac{(\sqrt{\delta\theta_f} + \sqrt{\theta_v})^2}{\delta[\mu - (1+\delta)\Lambda]} - \frac{R}{\delta} + T\}, & \text{otherwise.} \end{cases} \tag{EC.28}$$

As for the upper bound for Scenario VF, note that when $b = \bar{b}$, λ_v^{VF} just hits Λ. Thus, we have

$$\bar{b}_{VF} = \begin{cases} \min\{r_f, \frac{r_v}{\delta}\}, & \text{if } \mu \leq (1+\delta)\Lambda + \frac{\theta_v}{(1-\delta)R}, \\ \min\{r_f, \frac{r_v}{\delta}, \frac{\theta_f}{\mu - (1+\delta)\Lambda - \frac{\theta_v}{(1-\delta)R}} - R + T\}, & \text{otherwise.} \end{cases} \tag{EC.29}$$

We now move to deriving the lower bounds for the optimal b in each scenario; that is, the smallest b that enables achieving each scenario.

The smallest b that enables achieving Scenario NV is to make sure $\lambda_f^{NV} \geq 0$. Thus we have

$$\underline{b}_{NV} = \frac{\theta_f}{\mu} - R + T. \tag{EC.30}$$

The smallest b that enables achieving Scenario NF is to make sure $\lambda_v^{NF} \geq 0$. Thus we have

$$\underline{b}_{NF} = \frac{\theta_v}{\delta\Phi(\mu)} + \frac{\theta_f}{\mu - \Phi(\mu)} - \frac{R}{\delta} + T, \tag{EC.31}$$

where $\Phi(\mu)$ is defined in (EC.22).

The smallest b that enables achieving Scenario VF is to make sure $\lambda_v^{VF} \geq 0$. Thus we have

$$\underline{b}_{VF} = \begin{cases} \infty, \ \text{if } \mu \leq \Lambda + \frac{\theta_v}{(1-\delta)R}, \\ \frac{\theta_f}{\mu - \Lambda - \frac{\theta_v}{(1-\delta)R}} - R + T, \ \text{otherwise.} \end{cases} \tag{EC.32}$$

EC.1.3.2 Optimal Subsidy under Each Scenario

PROPOSITION EC.5 **(Optimal Subsidy b under Scenario NV)**.

$$b^{NV*} = \begin{cases} \underline{b}_{NV}, \ \textit{if } \tilde{b}^{NV} < \underline{b}_{NV}, \\ \tilde{b}^{NV}, \ \textit{if } \underline{b}_{NV} \leq \tilde{b}^{NV} \leq \bar{b}_{NV}, \\ \bar{b}_{NV}, \ \textit{if } \tilde{b}^{NV} > \bar{b}_{NV}, \end{cases}$$

where

$$\tilde{b}^{NV} = \sqrt{\frac{\theta_f(R - T + r_f)}{\mu}} - R + T.$$

PROPOSITION EC.6 **(Optimal Subsidy b under Scenario NF)**. *Define*

$$f(b) = \frac{\delta(\theta_f - \theta_v) + (1-\delta)\sqrt{\delta\theta_f\theta_v}}{R - \delta T + \delta b} - \frac{\theta_f}{R - T + b} + \frac{\delta\theta_v}{(1-\delta)R}.$$

Let

$$\tilde{b}^{NF} = \begin{cases} \frac{(\sqrt{\delta\theta_f} + \sqrt{\theta_v})\sqrt{R - \delta T + r_v}}{\delta\sqrt{\mu}} - \frac{R - \delta T}{\delta}, \ \textit{if } \frac{(\sqrt{\delta\theta_f} + \sqrt{\theta_v})\sqrt{R - \delta T + r_v}}{\delta\sqrt{\mu}} - \frac{R - \delta T}{\delta} \leq b^0 \\ \sqrt{\frac{\theta_f(\delta R - \delta T + r_v)}{\delta\mu - \frac{\delta\theta_v}{(1-\delta)R}}} - R + T, \ \textit{otherwise}, \end{cases}$$

where b^0 solves $f(b^0) = 0$. Suppose that $f(0) < 0$, then

$$b^{NF*} = \begin{cases} \underline{b}_{NF}, \ \textit{if } \tilde{b}^{NF} < \underline{b}_{NF}, \\ \tilde{b}^{NF}, \ \textit{if } \underline{b}_{NF} \leq \tilde{b}^{NF} \leq \bar{b}_{NF}, \\ \bar{b}_{NF}, \ \textit{if } \tilde{b}^{NF} > \bar{b}_{NF}. \end{cases}$$

The additional assumption $f(0) < 0$ restricts δ, the probability of needing a supplementary visit, from being too large. That aligns with practical return probabilities, which are estimated to be 6–20% (Yamamoto 2014, Uscher-Pines et al. 2016, Shi et al. 2018).

PROPOSITION EC.7 **(Optimal Subsidy b under Scenario VF).** *Define*

$$\tilde{b}^{VF} = \sqrt{\frac{\theta_f(r_v - r_f) - (1-\delta)\theta_f(R-T)}{-(1-\delta)\mu + \Lambda + \frac{\theta_v}{R}}} - R + T.$$

If $r_v - r_f - (1-\delta)(R-T) \leq 0$, $(1-\delta)\mu - \Lambda - \theta_v R^{-1} > 0$, and $\tilde{b}^{VF} \in [\underline{b}_{VF}, \bar{b}_{VF}]$, then $b^{VF*} = \tilde{b}^{VF}$. Otherwise, b^{VF*} takes the value of $\bar{b}_{VF}$ or $\underline{b}_{VF}$ whichever yields the higher net revenue.

EC.1.3.3 Thresholds for μ

The following analysis narrows the options for the optimal scenario under a given total service capacity μ. Specifically, it is the capacity needed for a specific scenario, when b is set to be the profit margin.

The smallest capacity required to assure Scenario NV is defined by letting μ achieve Region BF when $b = r_f$, i.e.,

$$\underline{\mu}_{NV} = \frac{\theta_f}{R - T + r_f}.$$

The smallest capacity required to assure Scenario NF is defined by letting μ achieve Region BV when $b = r_v\delta^{-1}$, i.e., $\underline{\mu}_{NF} = \underline{\mu}^{BV}(r_v\delta^{-1})$, where $\underline{\mu}^{BV}(r_v\delta^{-1})$ is similar to $\underline{\mu}^{BV}$ which is defined in E-Companion EC.1.2. Specifically, $\underline{\mu}_{NF}$ is achieved by solving the following equation for μ:

$$R - \delta T + r_v - \frac{\theta_v}{\Phi(\mu)} - \frac{\delta\theta_f}{\mu - \Phi(\mu)} = 0,$$

where $\Phi(\mu)$ is defined in (EC.22).

The smallest capacity required to assure Scenario VF is defined by letting μ achieve Region VF when $b = \min\{r_f, r_v\delta^{-1}\}$, i.e.,

$$\underline{\mu}_{VF} = \Lambda + \frac{\theta_f}{R - T + \min\{r_f, \frac{r_v}{\delta}\}} + \frac{\theta_v}{(1-\delta)R}.$$

We also define

$$\bar{\mu} = (1+\delta)\Lambda + \frac{\theta_f}{R-T} + \frac{\theta_v}{(1-\delta)R}.$$

When $\mu \geq \bar{\mu}$, the provider is able to achieve Regions V and F without any subsidy. In other words, there is no need to consider the use of transportation subsidy if $\mu \geq \bar{\mu}$.

EC.2 Incorporating Heterogeneity

We consider that the travel cost t is a random variable following a uniform distribution and normalize its scale such that $t \sim U[0,1]$. This normalization does not affect the results as other model parameters can be rescaled accordingly. For notational convenience, we present the analyses under $b=0$; the case with $b \geq 0$ can be analyzed in a similar fashion.

Suppose that the effective arrival rates are (λ_f, λ_v). For any $t \in [0,1]$, we have

$$U_f^t(\lambda_f, \lambda_v) = R - t - \frac{\theta_f}{\mu_f - \lambda_f - \delta\lambda_v},$$

$$U_v^t(\lambda_f, \lambda_v) = R - \delta t - \frac{\delta\theta_f}{\mu_f - \lambda_f - \delta\lambda_v} - \frac{\theta_f}{\mu_v - \lambda_v},$$

$$U_b^t(\lambda_f, \lambda_v) = 0.$$

It is straightforward to have the following Lemma.

LEMMA EC.2.

- *If $U_f^t(\lambda_f, \lambda_v) \geq 0$ then $U_f^{t'}(\lambda_f, \lambda_v) > 0$ for any $t' < t$; and if $U_f^t(\lambda_f, \lambda_v) \leq 0$ then $U_f^{t'}(\lambda_f, \lambda_v) < 0$ for any $t' > t$.*
- *If $U_v^t(\lambda_f, \lambda_v) \geq 0$ then $U_v^{t'}(\lambda_f, \lambda_v) > 0$ for any $t' < t$; and if $U_v^t(\lambda_f, \lambda_v) \leq 0$ then $U_v^{t'}(\lambda_f, \lambda_v) < 0$ for any $t' > t$.*
- *If $U_f^t(\lambda_f, \lambda_v) \geq U_v^t(\lambda_f, \lambda_v)$, then $U_f^{t'}(\lambda_f, \lambda_v) > U_v^{t'}(\lambda_f, \lambda_v)$ for any $t' < t$; and if $U_f^t(\lambda_f, \lambda_v) \leq U_v^t(\lambda_f, \lambda_v)$, then $U_f^{t'}(\lambda_f, \lambda_v) < U_v^{t'}(\lambda_f, \lambda_v)$ for any $t' > t$.*

Following Lemma EC.2, Theorem EC.2 below establishes the existence and uniqueness of the equilibrium strategy for patients, given any service capacity allocation (μ_f, μ_v). We first define the following regions.

DEFINITION EC.3.

- Region B:

$$\mu_f \leq \frac{\theta_f}{R}, \tag{EC.33}$$

$$\frac{\delta\theta_f}{\mu_f} + \frac{\theta_v}{\mu_v} \geq R. \tag{EC.34}$$

- Region V:

$$\theta_v\mu_f - (1-\delta)\theta_f\mu_v \leq [\delta\theta_v - (1-\delta)\theta_f]\Lambda, \tag{EC.35}$$

$$\frac{\delta\theta_f}{\mu_f - \delta\Lambda} + \frac{\theta_v}{\mu_v - \Lambda} \leq R - \delta, \tag{EC.36}$$

$$\mu_f > \delta\Lambda, \tag{EC.37}$$

$$\mu_v > \Lambda. \tag{EC.38}$$

- Region F:

$$\mu_f \geq \frac{\theta_f}{R-1} + \Lambda, \tag{EC.39}$$

$$\frac{\theta_v}{\mu_v} - \frac{(1-\delta)\theta_f}{\mu_f - \Lambda} \geq 1-\delta. \tag{EC.40}$$

- Region BVF:

$$\mu_v > \frac{\theta_v}{(1-\delta)R}, \tag{EC.41}$$

$$\Lambda + \frac{2\delta\theta_v\Lambda}{(1-\delta)\left[R\Lambda - \delta\mu_v - \sqrt{(R\Lambda-\delta\mu_v)^2 + \frac{4\delta\theta_v\Lambda}{(1-\delta)}}\right]} + \frac{2\delta\theta_f\Lambda}{2\delta(\mu_f-\Lambda) + (1-\delta)\left[R\Lambda + \delta\mu_v - \sqrt{(R\Lambda-\delta\mu_v)^2 + \frac{4\delta\theta_v\Lambda}{(1-\delta)}}\right]}$$
$$> \frac{R\Lambda + \delta\mu_v - \sqrt{(R\Lambda-\delta\mu_v)^2 + \frac{4\delta\theta_v\Lambda}{(1-\delta)}}}{2\delta} \quad \text{for } (1-\delta)\left[R\Lambda + \delta\mu_v - \sqrt{(R\Lambda-\delta\mu_v)^2 + \frac{4\delta\theta_v\Lambda}{(1-\delta)}}\right] > 2\delta(\Lambda - \mu_f), \tag{EC.42}$$

and, when $(1-\delta)\theta_f - \delta\theta_v > 0$:

$$[(1-\delta)\theta_f - \delta\theta_v]\,R\Lambda - \delta\,[(1-\delta)\theta_f\mu_v - \theta_v\mu_f] - \frac{[(1-\delta)\theta_f - \delta\theta_v]^2\,\Lambda}{(1-\delta)(\mu_f - \delta\mu_v)} > 0 \quad \text{for } (1-\delta)\theta_f\mu_v - \theta_v\mu_f \geq 0, \tag{EC.43}$$

$$\mu_f > \delta\mu_v \quad \text{for } (1-\delta)\theta_f\mu_v - \theta_v\mu_f \geq 0; \tag{EC.44}$$

when $(1-\delta)\theta_f - \delta\theta_v < 0$:

$$[(1-\delta)\theta_f - \delta\theta_v]\,R\Lambda - \delta\,[(1-\delta)\theta_f\mu_v - \theta_v\mu_f] - \frac{[(1-\delta)\theta_f - \delta\theta_v]^2\,\Lambda}{(1-\delta)(\mu_f - \delta\mu_v)} > 0 \quad \text{for } \mu_f < \delta\mu_v, \tag{EC.45}$$

$$(1-\delta)\theta_f\mu_v - \theta_v\mu_f < 0. \tag{EC.46}$$

- Region BF:

$$\mu_f > \frac{\theta_f}{R} \tag{EC.47}$$

$$\mu_f < \frac{\theta_f}{R-1} + \Lambda, \tag{EC.48}$$

$$\mu_v \leq \frac{\theta_v}{(1-\delta)R}. \tag{EC.49}$$

- Region BV:

$$\frac{\delta\theta_f}{\mu_f} + \frac{\theta_v}{\mu_v} < R, \tag{EC.50}$$

$$\frac{\delta\theta_f}{\mu_f - \delta\Lambda} + \frac{\theta_v}{\mu_v - \Lambda} > R - \delta \quad \text{for } \mu_f > \delta\Lambda \text{ and } \mu_v > \Lambda, \tag{EC.51}$$

and, when $(1-\delta)\theta_f - \delta\theta_v > 0$:

$$[(1-\delta)\theta_f - \delta\theta_v]\,R\Lambda - \delta\,[(1-\delta)\theta_f\mu_v - \theta_v\mu_f] - \frac{[(1-\delta)\theta_f - \delta\theta_v]^2\,\Lambda}{(1-\delta)(\mu_f - \delta\mu_v)} \leq 0 \quad \text{for } \mu_f > \delta\mu_v, \tag{EC.52}$$

$$(1-\delta)\theta_f\mu_v - \theta_v\mu_f \geq 0; \tag{EC.53}$$

when $(1-\delta)\theta_f - \delta\theta_v < 0$:

$$[(1-\delta)\theta_f - \delta\theta_v]\, R\Lambda - \delta\,[(1-\delta)\theta_f\mu_v - \theta_v\mu_f] - \frac{[(1-\delta)\theta_f - \delta\theta_v]^2\,\Lambda}{(1-\delta)(\mu_f - \delta\mu_v)} \le 0 \quad \text{for } (1-\delta)\theta_f\mu_v < \theta_v\mu_f, \tag{EC.54}$$

$$\mu_f < \delta\mu_v \quad \text{for } (1-\delta)\theta_f\mu_v < \theta_v\mu_f. \tag{EC.55}$$

- Region VF:

$$\theta_v\mu_f - (1-\delta)\theta_f\mu_v > [\delta\theta_v - (1-\delta)\theta_f]\,\Lambda, \tag{EC.56}$$

$$\frac{\theta_v}{\mu_v} - \frac{(1-\delta)\theta_f}{\mu_f - \Lambda} < 1-\delta \quad \text{for } \mu_f > \Lambda, \tag{EC.57}$$

$$\Lambda + \frac{2\delta\theta_v\Lambda}{(1-\delta)\left[R\Lambda - \delta\mu_v - \sqrt{(R\Lambda-\delta\mu_v)^2 + \frac{4\delta\theta_v\Lambda}{(1-\delta)}}\right]} + \frac{2\delta\theta_f\Lambda}{2\delta(\mu_f - \Lambda) + (1-\delta)\left[R\Lambda + \delta\mu_v - \sqrt{(R\Lambda-\delta\mu_v)^2 + \frac{4\delta\theta_v\Lambda}{(1-\delta)}}\right]}$$
$$\le \frac{R\Lambda + \delta\mu_v - \sqrt{(R\Lambda-\delta\mu_v)^2 + \frac{4\delta\theta_v\Lambda}{(1-\delta)}}}{2\delta}, \tag{EC.58}$$

$$(1-\delta)\left[R\Lambda + \delta\mu_v - \sqrt{(R\Lambda-\delta\mu_v)^2 + \frac{4\delta\theta_v\Lambda}{(1-\delta)}}\right] > 2\delta(\Lambda - \mu_f). \tag{EC.59}$$

THEOREM EC.2. *The seven regions of (μ_f, μ_v) defined above are mutually exclusive and collectively exhaustive. And in each region, there exists a unique equilibrium strategy such that*

1. *When $(\mu_f, \mu_v) \in$ Region B, there exists a unique equilibrium strategy – strategy B – such that $\lambda_f = \lambda_v = 0$.*

2. *When $(\mu_f, \mu_v) \in$ Region V, there exists a unique equilibrium strategy – strategy V – such that $\lambda_v = \Lambda$.*

3. *When $(\mu_f, \mu_v) \in$ Region F, there exists a unique equilibrium strategy – strategy F – such that $\lambda_f = \Lambda$.*

4. *When $(\mu_f, \mu_v) \in$ Region BVF, there exists a unique equilibrium strategy – strategy BVF – such that $\lambda_f + \lambda_v \le \Lambda$.*

The effective arrival rates (λ_f, λ_v) are

$$\lambda_f = \tfrac{1}{2}\left[\mu_f - \delta\mu_v - \sqrt{(R\Lambda-\mu_f)^2 + 4\theta_f\Lambda} + \delta\sqrt{\left(\frac{R\Lambda}{\delta} - \mu_v\right)^2 + \frac{4\theta_v\Lambda}{\delta(1-\delta)}}\right]$$
$$\lambda_v = \tfrac{1}{2}\left[\frac{R\Lambda}{\delta} + \mu_v - \sqrt{\left(\frac{R\Lambda}{\delta} - \mu_v\right)^2 + \frac{4\theta_v\Lambda}{\delta(1-\delta)}}\right].$$

5. *When $(\mu_f, \mu_v) \in$ Region BF, there exists a unique equilibrium strategy – strategy BF – such that $\lambda_f \le \Lambda$, $\lambda_v = 0$. The effective arrival rate λ_f is*

$$\lambda_f = \tfrac{1}{2}\left[R\Lambda + \mu_f - \sqrt{(R\Lambda-\mu_f)^2 + 4\theta_f\Lambda}\right].$$

6. *When $(\mu_f, \mu_v) \in$ Region BV, there exists a unique equilibrium strategy – strategy BV – such that $\lambda_f = 0$, $\lambda_v \le \Lambda$. The effective arrival rate λ_v can be derived by solving*

$$\frac{R\Lambda}{\delta} - \frac{\theta_v \Lambda}{\delta(\mu_v - \lambda_v)} - \frac{\theta_f \Lambda}{\mu_f - \delta\lambda_v} = \lambda_v$$

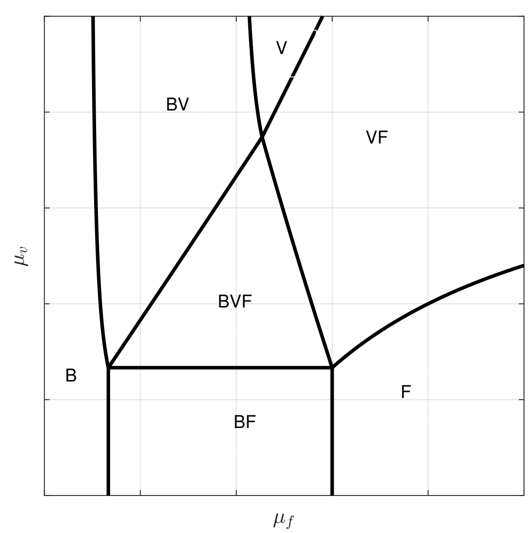
7. *When $(\mu_f, \mu_v) \in$ Region VF, there exists a unique equilibrium strategy – strategy VF – such that $\lambda_f + \lambda_v = \Lambda$. The effective arrival rates (λ_f, λ_v) can be derived from solving the equations $\lambda_f + \lambda_v = \Lambda$ and*

$$\Lambda - \frac{\theta_v \Lambda}{(1-\delta)(\mu_v - \lambda_v)} + \frac{\theta_f \Lambda}{\mu_f - \Lambda + (1-\delta)\lambda_v} = \lambda_v.$$

Theorem EC.2 establishes that all possible capacity allocations fall into seven mutually exclusive and collectively exhaustive cases. Letting the x-axis represent μ_f and the y-axis represent μ_v, Figure EC.1 illustrates these seven regions.

Although the formulations are more complex in the case of heterogeneous travel costs, the structure of the seven regions remains very similar to that described in Theorem EC.1.

Figure EC.1 Illustration of the seven equilibrium regions in the case of uniformly distributed travel costs.



EC.3 Proofs of Analytical Results

EC.3.1 Proofs of Analytical Results in Section 3 and E-Companion EC.1.1

Proof of Theorem EC.1 (extended version of Theorem 1) For notational convenience, we carry out the proof under $b = 0$. The results extend directly to any $b \geq 0$ by replacing T with $T - b$ throughout.

By Definition EC.1, it is easy to see (EC.1) v.s. (EC.15) divide the (μ_f, μ_f) space; (EC.2) v.s. (EC.12) divide the (μ_f, μ_f) space; (EC.3) with (EC.6) and v.s. (EC.19) with (EC.21) divide the (μ_f, μ_f) space; (EC.4) with (EC.5) and (EC.6) v.s. (EC.16) divide the (μ_f, μ_f) space; (EC.7) v.s. (EC.13) divide the (μ_f, μ_f) space; (EC.8) with $\mu_f \geq \Lambda$ v.s. (EC.18) divide the (μ_f, μ_f) space; (EC.9) v.s. (EC.14) divide the (μ_f, μ_f) space; (EC.10) v.s. (EC.17) divide the (μ_f, μ_f) space; (EC.11) v.s. (EC.20) divide the (μ_f, μ_f) space. Thus the seven regions of (μ_f, μ_v) defined in Definition EC.1 are mutually exclusive and collectively exhaustive.

We conduct the analysis for each region separately: Region B: Since

$$\frac{\theta_v}{\mu_v} + \frac{\delta\theta_f}{\mu_f} \geq R - \delta T \quad \text{and} \quad \mu_f \leq \frac{\theta_f}{R - T},$$

we get that $U_f(0,0) \leq 0$ and $U_v(0,0) \leq 0$ and therefore, there exists a unique equilibrium such that $\lambda_f = \lambda_v = 0$.

Region V: Since

$$\frac{\theta_v}{\mu_v - \Lambda} - \frac{(1-\delta)\theta_f}{\mu_f - \delta\Lambda} \leq (1-\delta)T, \quad \frac{\theta_v}{\mu_v - \Lambda} + \frac{\delta\theta_f}{\mu_f - \delta\Lambda} \leq R - \delta T,$$
$$\mu_f \geq \delta\Lambda, \quad \text{and} \quad \mu_v \geq \Lambda,$$

we get that $U_f(0,\Lambda) \leq U_v(0,\Lambda)$ and $U_v(0,\Lambda) \geq 0$ and therefore, there exists a unique equilibrium such that $\lambda_f = 0$ and $\lambda_v = \Lambda$.

Region F: Since

$$\mu_f \geq \Lambda + \frac{\theta_f}{R - T} \quad \text{and} \quad \frac{\theta_v}{\mu_v} - \frac{(1-\delta)\theta_f}{\mu_f - \Lambda} \geq (1-\delta)T,$$

we get that $U_f(\Lambda, 0) \geq U_v(\Lambda, 0)$ and $U_f(\Lambda, 0) \geq 0$ and therefore, there exists a unique equilibrium such that $\lambda_f = \Lambda$ and $\lambda_v = 0$.

Region BVF: The equality $U_v(\lambda_f, \lambda_v) = U_f(\lambda_f, \lambda_v) = 0$ yields the following conditions.

$$\lambda_v = \mu_v - \frac{\theta_v}{(1-\delta)R} \quad \text{and} \quad \lambda_f + \delta\lambda_v = \mu_f - \frac{\theta_f}{R - T}.$$

Then we have

$$\lambda_f = \mu_f - \frac{\theta_f}{R - T} - \delta\mu_v + \frac{\delta\theta_v}{(1-\delta)R}.$$

The constraints of Region BVF:

$$\mu_v \geq \frac{\theta_v}{(1-\delta)R},\ \ \mu_f - \delta\mu_v \geq \frac{\theta_f}{R-T} - \frac{\delta\theta_v}{(1-\delta)R},\ \ \mu_f + (1-\delta)\mu_v \leq \Lambda + \frac{\theta_f}{R-T} + \frac{\theta_v}{R},$$

guarantee $\lambda_v \geq 0$, $\lambda_f \geq 0$ and $\lambda_v + \lambda_f \leq \Lambda$. Thus, there exists a unique equilibrium such that $\lambda_v + \lambda_f \leq \Lambda$.

Region BF: The fact that $U_f(\lambda_f, 0) = 0$ yields the following result:

$$\lambda_f = \mu_f - \frac{\theta_f}{R-T}.$$

The constraints of Region BF:

$$\frac{\theta_f}{R-T} \leq \mu_f \leq \Lambda + \frac{\theta_f}{R-T},\ \ \text{and}\ \ \mu_v \leq \frac{\theta_v}{(1-\delta)R}$$

guarantee $0 \leq \lambda_f \leq \Lambda$ and $U_v(\lambda_f, 0) \leq 0$. Thus, there exists a unique equilibrium such that $0 \leq \lambda_f \leq \Lambda$ and $\lambda_v = 0$.

Region BV: The fact that $U_v(0, \lambda_v) = 0$ yields the following conditions:

$$R - \delta T - \frac{\theta_v}{\mu_v - \lambda_v} - \frac{\delta\theta_f}{\mu_f - \delta\lambda_v} = 0, \tag{EC.60}$$

$$\mu_v - \lambda_v > 0\ \ \text{and}\ \ \mu_f - \delta\lambda_v > 0.$$

Next, we show that

1) There is only one λ_v which solves (EC.60) and satisfies $\mu_f - \delta\lambda_v > 0$ and $\mu_v - \lambda_v > 0$.

2) With the constraints of Region BV, we must have $0 \leq \lambda_v \leq \Lambda$ and $U_f(0, \lambda_v) \leq 0$.

Thus, there exists a unique equilibrium such that $0 \leq \lambda_v \leq \Lambda$ and $\lambda_f = 0$.

We start with proving 1): There is only one λ_v which solves (EC.60) and satisfies $\mu_f - \delta\lambda_v > 0$ and $\mu_v - \lambda_v > 0$.

Specifically, by solving (EC.60), we have

$$\lambda_v = \frac{\mathcal{B} - \sqrt{\mathcal{D}^2 + 4\delta^2\theta_f\theta_v}}{2\mathcal{A}}, \tag{EC.61}$$

where

$$\mathcal{A} = (R - \delta T)\delta,\ \ \mathcal{B} = (R - \delta T)(\mu_f + \delta\mu_v) - \delta(\theta_f + \theta_v),$$

$$\mathcal{C} = (R - \delta T)\mu_f\mu_v - \theta_v\mu_f - \delta\theta_f\mu_v,\ \ \mathcal{D} = (R - \delta T)(\mu_f - \delta\mu_v) - \delta(\theta_f - \theta_v).$$

Let us rewrite (EC.60) as

$$(R - \delta T)\delta\lambda_v^2 - [(R - \delta T)(\mu_f + \delta\mu_v) - \delta(\theta_f + \theta_v)]\lambda_v + [(R - \delta T)\mu_f\mu_v - \theta_v\mu_f - \delta\theta_f\mu_v] = 0,$$

or alternatively, $\mathcal{A}\lambda_v^2 - \mathcal{B}\lambda_v + \mathcal{C} = 0$. Note that

$$\mathcal{B}^2 - 4\mathcal{A}\mathcal{C} = [(R-\delta T)(\mu_f - \delta\mu_v) - \delta(\theta_f - \theta_v)]^2 + 4\delta^2\theta_f\theta_v = \mathcal{D}^2 + 4\delta^2\theta_f\theta_v \geq 0.$$

Then, (EC.60) has two possible solutions:

$$\lambda_v^{BV.1} = \frac{\mathcal{B} + \sqrt{\mathcal{D}^2 + 4\delta^2\theta_f\theta_v}}{2\mathcal{A}} \quad \text{and} \quad \lambda_v^{BV.2} = \frac{\mathcal{B} - \sqrt{\mathcal{D}^2 + 4\delta^2\theta_f\theta_v}}{2\mathcal{A}}.$$

There are also two cases for $\mathcal{D}$:

- **Case 1:** $\mathcal{D} > 0$

—If $\lambda_v = \lambda_v^{BV.1}$,

$$\lambda_v^{BV.1} > \frac{\mathcal{B}+\mathcal{D}}{2\mathcal{A}} = \frac{(R-\delta T)\mu_f - \delta\theta_f}{(R-\delta T)\delta}.$$

Then, we have $\mu_f - \delta\lambda_v^{BV.1} < \frac{\delta\theta_f}{R-\delta T}$, which contradicts with either (EC.60) or $\mu_f - \delta\lambda_v > 0$.

—If $\lambda_v = \lambda_v^{BV.2}$,

$$\lambda_v^{BV.2} < \frac{\mathcal{B}-\mathcal{D}}{2\mathcal{A}} = \frac{(R-\delta T)\mu_v - \theta_v}{(R-\delta T)}.$$

Then, we have $\mu_v - \lambda_v^{BV.2} > \frac{\theta_v}{R-\delta T} > 0$, and $\lambda_v^{BV.2} < \frac{\mu_f}{\delta}$ (since $\mathcal{D} > 0$), which validates (EC.60), $\mu_v > \lambda_v$ and $\mu_f - \delta\lambda_v > 0$.

- **Case 2:** $\mathcal{D} \leq 0$

—If $\lambda_v = \lambda_v^{BV.1}$,

$$\lambda_v^{BV.1} > \frac{\mathcal{B}-\mathcal{D}}{2\mathcal{A}} = \frac{(R-\delta T)\mu_v - \theta_v}{(R-\delta T)}.$$

Then, we have $\mu_v - \lambda_v^{BV.1} < \frac{\theta_v}{R-\delta T}$, which contradicts with (EC.60) or $\lambda_v < \mu_v$.

—If $\lambda_v = \lambda_v^{BV.2}$,

$$\lambda_v^{BV.2} < \frac{\mathcal{B}+\mathcal{D}}{2\mathcal{A}} = \frac{(R-\delta T)\mu_f - \delta\theta_f}{(R-\delta T)\delta}.$$

Then, we have $\mu_f - \delta\lambda_v^{BV.2} > \frac{\delta\theta_f}{R-\delta T} > 0$, and $\lambda_v^{BV.2} < \mu_v$ (by $\mathcal{D} \leq 0$), which validates (EC.60), $\mu_v > \lambda_v$ and $\mu_f - \delta\lambda_v > 0$.

In sum, $\lambda_v = \lambda_v^{BV.2}$ is the only solution which solves (EC.60) and satisfies $\mu_f - \delta\lambda_v > 0$ and $\mu_v - \lambda_v > 0$.

Next, we move on to proving 2): With the constraints of Region BV, we must have $0 \leq \lambda_v \leq \Lambda$ and $U_f(0, \lambda_v) \leq 0$; specifically, we prove the following:

2.1) $\lambda_v \geq 0$. Recall the constraints of Region BV:

$$\frac{\theta_v}{\mu_v} + \frac{\delta\theta_f}{\mu_f} \leq R - \delta T, \quad \mu_f - \delta\mu_v \leq \frac{\theta_f}{R-T} - \frac{\delta\theta_v}{(1-\delta)R},$$

$$\frac{\theta_v}{\mu_v - \Lambda} + \frac{\delta\theta_f}{\mu_f - \delta\Lambda} \geq R - \delta T \text{ for } \mu_v > \Lambda \text{ and } \mu_f > \delta\Lambda.$$

Then we have $\mathcal{A} > 0$, $\mathcal{B} > 0$ and $\mathcal{C} \geq 0$. Since $\mathcal{B}^2 - 4\mathcal{A}\mathcal{C} \geq 0$, then we have $\mathcal{B} - \sqrt{\mathcal{B}^2 - 4\mathcal{A}\mathcal{C}} \geq 0$. Thus $\lambda_v \geq 0$.

2.2) $\lambda_v \leq \Lambda$. If $\mu_v \leq \Lambda$ or $\mu_f \leq \delta\Lambda$, then $\lambda_v < \Lambda$ since $\mu_v > \lambda_v$ and $\mu_f > \delta\lambda_v$.

If $\mu_v > \Lambda$ and $\mu_f > \delta\Lambda$, by (EC.60) and the constraint

$$\frac{\theta_v}{\mu_v - \Lambda} + \frac{\delta\theta_f}{\mu_f - \delta\Lambda} \geq R - \delta T \text{ for } \mu_v > \Lambda \text{ and } \mu_f > \delta\Lambda,$$

we must have $\lambda_v \leq \Lambda$.

2.3) $U_f(0, \lambda_v) \leq 0$. By (EC.60) we have

$$\delta(R - T - \frac{\theta_f}{\mu_f - \delta\lambda_v}) = \frac{\theta_v}{\mu_v - \lambda_v} - (1-\delta)R.$$

Suppose

$$U_f(0, \lambda_v) = R - T - \frac{\theta_f}{\mu_f - \delta\lambda_v} > 0,$$

we must have

$$\frac{\theta_v}{\mu_v - \lambda_v} - (1-\delta)R > 0.$$

Then we should have

$$\mu_f - \delta\lambda_v > \frac{\theta_f}{R - T}$$

and

$$\mu_v - \lambda_v < \frac{\theta_v}{(1-\delta)R},$$

which contradict with

$$\mu_f - \delta\mu_v \leq \frac{\theta_f}{R - T} - \frac{\delta\theta_v}{(1-\delta)R}.$$

Thus we must have $U_f(0, \lambda_v) \leq 0$.

Region VF: The fact that $\lambda_v + \lambda_f = \Lambda$ and $U_f(\lambda_f, \lambda_v) = U_v(\lambda_f, \lambda_v)$ yield the following conditions

$$\frac{\theta_v}{\mu_v - \lambda_v} - \frac{(1-\delta)\theta_f}{\mu_f - \Lambda + (1-\delta)\lambda_v} = (1-\delta)T, \quad \text{(EC.62)}$$

$$\frac{\Lambda - \mu_f}{1-\delta} < \lambda_v < \mu_v.$$

Recall the constraints of Region VF:

$$\begin{aligned}
&\frac{\theta_v}{\mu_v} - \frac{(1-\delta)\theta_f}{\mu_f - \Lambda} \leq (1-\delta)T \text{ for } \mu_f > \Lambda, \\
&\frac{\theta_v}{\mu_v - \Lambda} - \frac{(1-\delta)\theta_f}{\mu_f - \delta\Lambda} \geq (1-\delta)T \text{ for } \mu_v > \Lambda, \\
&\mu_f + (1-\delta)\mu_v \geq \Lambda + \frac{\theta_f}{R - T} + \frac{\theta_v}{R}, \text{ and} \\
&\mu_f \geq \delta\Lambda.
\end{aligned}$$

Since the left-hand side of (EC.62) is increasing in λ_v when $(\Lambda-\mu_f)/(1-\delta)<\lambda_v<\mu_v$. By the first two constraints of Region VF, there must be a unique λ_v such that $0\le\lambda_v\le\Lambda$ solving (EC.62). Suppose that $U_v(\Lambda-\lambda_v,\lambda_v)<0$, by (EC.62) we have

$$U_f(\Lambda-\lambda_v,\lambda_v)=R-T-\frac{\theta_f}{\mu_f-\Lambda+(1-\delta)\lambda_v}<0$$

and

$$(1-\delta)R-\frac{\theta_v}{\mu_v-\lambda_v}<0.$$

Then we should have

$$\mu_f+(1-\delta)\mu_v<\Lambda+\frac{\theta_f}{R-T}+\frac{\theta_v}{R}$$

which contradicts with the third constraint of Region VF. Thus we must have $U_f(\Lambda-\lambda_v,\lambda_v)=U_v(\Lambda-\lambda_v,\lambda_v)\ge 0$.

Thus, there exists a unique equilibrium such that $\lambda_f+\lambda_v=\Lambda$. Specifically, solving (EC.62) gives

$$\lambda_v=\frac{-\tilde{\mathcal{B}}+\sqrt{\tilde{\mathcal{D}}^2+4\theta_f\theta_v}}{2\tilde{\mathcal{A}}},\tag{EC.63}$$

where

$$\tilde{\mathcal{A}}=(1-\delta)T,\;\;\tilde{\mathcal{B}}=(\theta_f+\theta_v)+T(\mu_f-\Lambda-(1-\delta)\mu_v),$$
$$\tilde{\mathcal{C}}=\frac{\theta_v(\mu_f-\Lambda)}{(1-\delta)}-T\mu_v(\mu_f-\Lambda)-\theta_f\mu_v,\;\text{ and }\;\tilde{\mathcal{D}}=(\theta_f-\theta_v)+T(\mu_f-\Lambda+(1-\delta)\mu_v).$$

Q.E.D.

Proof of Lemma 1: For any feasible solution satisfying $\mu_f+\mu_v<\mu$, we can construct another feasible solution (μ_f',μ_v') such that $\mu_f+\mu_v<\mu_f'+\mu_v'\le\mu$, whose objective value is no smaller than that of (μ_f,μ_v). By Theorem EC.1, it is straightforward to verify the following cases:

- If (μ_f,μ_v) lies in Region B, V, F, BVF, or BF, then the solution with $\mu_f'=\mu_f+\varepsilon$ and $\mu_v'=\mu_v$ does not yield a lower objective value.
- If (μ_f,μ_v) lies in Region BV, then the solution with $\mu_f'=\mu_f$ and $\mu_v'=\mu_v+\varepsilon$ does not yield a lower objective value.
- If (μ_f,μ_v) lies in Region VF and $r_v>r_f$, then the solution with $\mu_f'=\mu_f$ and $\mu_v'=\mu_v+\varepsilon$ does not yield a lower objective value.
- If (μ_f,μ_v) lies in Region VF and $r_v\le r_f$, then the solution with $\mu_f'=\mu$ and $\mu_v'=0$ does not yield a lower objective value.

Therefore, increasing the total utilized capacity up to μ (weakly) improves the objective value, implying that the optimal solution must satisfy $\mu_f+\mu_v=\mu$. Q.E.D.

EC.3.2 Proofs of Analytical Results in Section 4 and E-Companion EC.1.2

Proof of Proposition EC.1: It directly follows Definition EC.1 and Theorem EC.1. Q.E.D.

Proof of Proposition EC.2: Recall that in Region BVF, $0 \le \lambda_f + \lambda_v \le \Lambda$. The provider's objective is to maximize $r_f\lambda_f + r_v\lambda_v$. Since in this region we have

$$\lambda_v = \mu_v - \frac{\theta_v}{(1-\delta)R}, \quad \text{and} \quad \lambda_f = \mu_f - \delta\mu_v - \frac{\theta_f}{R-T} + \frac{\theta_v}{(1-\delta)R},$$

the objective function can be rewritten as

$$r_f\left(\mu_f - \delta\mu_v - \frac{\theta_f}{R-T} + \frac{\theta_v}{(1-\delta)R}\right) + r_v\left(\mu_v - \frac{\theta_v}{(1-\delta)R}\right),$$

which by substituting $\mu_v = \mu - \mu_f$ and removing the constants is equivalent to

$$[(1+\delta)r_f - r_v]\,\mu_f.$$

Therefore, the provider's problem is

$$\begin{aligned} \max_{0\le\mu_f\le\mu} \quad & [(1+\delta)r_f - r_v]\,\mu_f \\ s.t. \quad & \mu - \mu_f \ge \frac{\theta_v}{(1-\delta)R}, \\ & \mu_f - \delta(\mu-\mu_f) \ge \frac{\theta_f}{R-T} - \frac{\delta\theta_v}{(1-\delta)R}, \\ & \mu_f + (1-\delta)(\mu-\mu_f) \le \Lambda + \frac{\theta_f}{R-T} + \frac{\theta_v}{R}. \end{aligned} \tag{EC.64}$$

The optimal solution then depends on the sign of the objective function and the boundaries of μ_f in (EC.64). Q.E.D.

Proof of Proposition EC.3: Before characterizing the optimal capacity allocation in Region BV, we first provide an auxiliary result. Let $\lambda_v^{BV}(\mu-\mu_v,\mu_v)$ denote the unique effective arrival rate to the virtual channel (which is defined in (EC.61) and here we denote it as λ_v^{BV}) by substituting μ_f by $\mu-\mu_v$. Lemma EC.3 proves that $\lambda_v^{BV}(\mu-\mu_v,\mu_v)$ is a unimodal function in μ_v, which is maximized at the unique value $\mu_v = \tilde{\mu}_v^{BV}$.

LEMMA EC.3. *Given (μ_f,μ_v) which satisfies (EC.60) and (EC.15)–(EC.17), there exists a unique $\tilde{\mu}_v^{BV}$ such that $\lambda_v^{BV}(\mu-\mu_v,\mu_v)$ is increasing in μ_v when $\mu_v \le \tilde{\mu}_v^{BV}$ and $\lambda_v^{BV}(\mu-\mu_v,\mu_v)$ is decreasing in μ_v when $\mu_v \ge \tilde{\mu}_v^{BV}$.*

In Proposition EC.3, when

$$\mu \ge \Lambda(1+\delta) + \min\left\{\frac{\theta_f}{R-T} + \frac{\theta_v}{(1-\delta)R}, \frac{(\sqrt{\delta\theta_f}+\sqrt{\theta_v})^2}{R-\delta T}\right\},$$

the results are straightforward as Λ can be achieved.

When

$$\mu < \Lambda(1+\delta) + \min\left\{\frac{\theta_f}{R-T} + \frac{\theta_v}{(1-\delta)R}, \frac{(\sqrt{\delta\theta_f}+\sqrt{\theta_v})^2}{R-\delta T}\right\}, \tag{EC.65}$$

we observe that the optimal solution is $\tilde{\mu}_v^{BV}$ without constraints (EC.15)–(EC.17) (it directly follows from Lemma EC.3). Next, we have to regularize μ_v^{BV*} to make (EC.15)–(EC.17) valid. Recall the constraints (EC.15)–(EC.17):

$$\frac{\theta_v}{\mu_v} + \frac{\delta\theta_f}{\mu_f} \le R - \delta T, \ \ \mu_f - \delta\mu_v \le \frac{\theta_f}{R-T} - \frac{\delta\theta_v}{(1-\delta)R},$$

$$\frac{\theta_v}{\mu_v - \Lambda} + \frac{\delta\theta_f}{\mu_f - \delta\Lambda} \ge R - \delta T \text{ for } \mu_v > \Lambda \text{ and } \mu_f > \delta\Lambda.$$

Note that $\tilde{\mu}_v^{BV}$ must satisfy the first condition, otherwise no revenue is generated; $\tilde{\mu}_v^{BV}$ must satisfy the third condition, by (EC.65). When $(\mu - \tilde{\mu}_v^{BV}, \tilde{\mu}_v^{BV}) \in$ Region BV, the second constraint is also satisfied, then $(\mu - \tilde{\mu}_v^{BV}, \tilde{\mu}_v^{BV})$ is the optimal solution, yielding

$$\lambda_v^{BV*} = \frac{\mu}{1+\delta} - \frac{(\sqrt{\delta\theta_f}+\sqrt{\theta_v})^2}{(1+\delta)(R-\delta T)}.$$

When $(\mu - \tilde{\mu}_v^{BV}, \tilde{\mu}_v^{BV}) \notin$ Region BV, the second constraint is not satisfied, i.e, the boundary solution would be the optimal one:

$$\frac{\mu}{(1+\delta)} - \frac{\theta_f}{(1+\delta)(R-T)} + \frac{\delta\theta_v}{(1+\delta)(1-\delta)R},$$

yielding

$$\lambda_v^{BV*} = \frac{\mu}{1+\delta} - \frac{\theta_f}{(1+\delta)(R-T)} - \frac{\theta_v}{(1+\delta)(1-\delta)R}.$$

Q.E.D.

Proof of Lemma EC.3: Recall that given (μ_f, μ_v) which satisfies (EC.60) and (EC.15)–(EC.17), we have

$$\lambda_v^{BV} = \frac{\mathcal{B} - \sqrt{\mathcal{D}^2 + 4\delta^2\theta_f\theta_v}}{2\mathcal{A}}.$$

By replacing μ_f with $\mu - \mu_v$, we have

$$\frac{\partial\mathcal{B}}{\partial\mu_v} = -(R-\delta T)(1-\delta) < 0, \ \text{ and } \ \frac{\partial\mathcal{D}}{\partial\mu_v} = -(R-\delta T)(1+\delta) < 0.$$

Therefore, when $\mathcal{D} < 0$, λ_v^{BV} is decreasing in μ_v. Note that $\mathcal{A}$ is independent of μ_v.

When $\mathcal{D} \ge 0$,

$$\frac{\partial\lambda_v^{BV}}{\partial\mu_v} = -\frac{(1-\delta)}{2\delta} + \frac{(1+\delta)\mathcal{D}}{2\delta\sqrt{\mathcal{D}^2 + 4\delta^2\theta_f\theta_v}}.$$

Since $\mathcal{D} \ge 0$, then $\frac{\partial\lambda_v^{BV}}{\partial\mu_v}$ is increasing in $\mathcal{D}$. And when $\mathcal{D} = 0$, $\frac{\partial\lambda_v^{BV}}{\partial\mu_v} = -\frac{(1-\delta)}{2\delta} < 0$. Let $\bar{D}$ solves $\frac{\partial\lambda_v^{BV}}{\partial\mu_v} = 0$, we must have $\bar{D} > 0$. So when $0 \le \mathcal{D} < \bar{D}$, $\frac{\partial\lambda_v^{BV}}{\partial\mu_v} < 0$; and when $\mathcal{D} > \bar{D}$, $\frac{\partial\lambda_v^{BV}}{\partial\mu_v} > 0$.

When

$$\mu_v = \tilde{\mu}_v^{BV} = \frac{\mu}{1+\delta} - \frac{\delta(\theta_f - \theta_v) + (1-\delta)\sqrt{\delta\theta_f\theta_v}}{(1+\delta)(R-\delta T)},$$

$\mathcal{D} = \bar{D}$. Since $\mathcal{D}$ is decreasing in μ_v, we can conclude that when $\mu_v \leq \tilde{\mu}_v^{BV}$, λ_v^{BV} is increasing in μ_v; when $\mu_v \geq \tilde{\mu}_v^{BV}$, λ_v^{BV} is decreasing in μ_v. Q.E.D.

Proof of Proposition EC.4: Before characterizing the optimal capacity allocation in Region VF, we first provide an auxiliary result. Let $\lambda_v^{VF}(\mu-\mu_v, \mu_v)$ denote the unique effective arrival rate to the virtual channel (which is defined in (EC.63) and here we denote it as λ_v^{VF}) by substituting μ_f by $\mu - \mu_v$. Lemma EC.4 proves that $\lambda_v^{VF}(\mu-\mu_v, \mu_v)$ is increasing in μ_v.

LEMMA EC.4. *Given (μ_f, μ_v) which satisfies (EC.62) and (EC.18)–(EC.21), λ_v^{VF} is increasing in μ_v.*

Since $\lambda_v^{VF}(\mu-\mu_v, \mu_v)$ is increasing in μ_v and the objective function is monotone in λ_v^{VF}, the optimal solution must be on the boundaries if $r_v \neq r_f$. Q.E.D.

Proof of Lemma EC.4: Recall that given (μ_f, μ_v) which satisfies (EC.62) and (EC.18)–(EC.21), we have

$$\lambda_v^{VF} = \frac{-\tilde{\mathcal{B}} + \sqrt{\tilde{\mathcal{D}}^2 + 4\theta_f\theta_v}}{2\tilde{\mathcal{A}}}.$$

By replacing μ_f with $\mu - \mu_v$, we have

$$\frac{\partial \tilde{\mathcal{B}}}{\partial \mu_v} = -(2-\delta)T < 0, \quad \text{and} \quad \frac{\partial \tilde{\mathcal{D}}}{\partial \mu_v} = -\delta T < 0.$$

Then,

$$\frac{\partial(-\tilde{\mathcal{B}} + \sqrt{\tilde{\mathcal{D}}^2 + 4\theta_f\theta_v})}{\partial \mu_v} = (2-\delta)T - \frac{\delta T \tilde{\mathcal{D}}}{\sqrt{\mathcal{D}^2 + 4\theta_f\theta_v}},$$

which is decreasing in $\tilde{\mathcal{D}}$. When $\tilde{\mathcal{D}} \to \infty$,

$$\frac{\partial(-\tilde{\mathcal{B}} + \sqrt{\tilde{\mathcal{D}}^2 + 4\theta_f\theta_v})}{\partial \mu_v} \to 2(1-\delta)T > 0.$$

So $\lambda_v^{VF}(\mu-\mu_v, \mu_v)$ is increasing in μ_v. Q.E.D.

Proof of Theorem 2: As illustrated in Figure 3, each system size includes several equilibrium regions. The optimal capacity allocation for each size is the one that yields the greatest profit among the regions it includes.

1. **Small System** includes Regions B, BV, BVF, BF and F. Region B is dominated by Regions BV and BF. From Proposition EC.2 we know that the optimal solution in Region BVF is on the boundaries (in this case, either on the boundary with BV or on the boundary with BF). When Region F is applicable ($\Lambda \leq \mu - \frac{\theta_f}{R-T}$), it dominates Region BF. The optimal value function is $r_f \Lambda$ in Region F, and $r_f \left(\mu - \frac{\theta_f}{R-T}\right)$ in Region BF. In both cases the optimal allocation is $\mu_f = \mu$ and $\mu_v = 0$. The optimal value function in Regions F and BF is, therefore, $r_f \left(\min\{\Lambda, \mu - \frac{\theta_f}{R-T}\}\right)$. Comparing the latter with the optimal value function in Region BV yields the condition as stated.

2. **Medium-sized System** includes Regions B, BV, BVF, VF and F. Region B is dominated by Region BV. From Proposition EC.2 we know that the optimal solution in Region BVF is dominated by either BV or VF. Moreover, Region F is dominated by VF (as an extreme case in VF). Therefore, it suffices to compare the optimal value function in Regions BV and VF, as stated.

3. **Large System** includes Regions B, BV, V, VF and F. Region B is dominated by Region BV and Regions BV and VF are dominated by Region V (as an extreme case in VF). Therefore, the optimal region is VF, as stated. Since the solution of Region VF is a boundary one, it will utilize either the in-person channel or the virtual channel while supporting returning patients. Q.E.D.

EC.3.3 Proofs of Analytical Results in Section 5 and E-Companion EC.1.3

Proof of Corollary EC.1: First let us replace T by $T - b$ in the results of Theorem 2. Then, the optimal capacity allocation must lead to the six scenarios presented in the Corollary statement. The results in each scenario are straightforward. It is worth noting that

$$\frac{\delta(\theta_f - \theta_v) + (1-\delta)\sqrt{\delta\theta_f\theta_v}}{R - \delta T + \delta b} \leq \frac{\theta_f}{R - T + b} - \frac{\delta\theta_v}{(1-\delta)R}$$

is equivalent to the second constraint for $(\mu - \mu_v, \mu_v)$ in Proposition EC.3, i.e., $(\mu - \tilde{\mu}_v^{BV}, \tilde{\mu}_v^{BV}) \in$ Region BV. Q.E.D.

Proof of Lemma EC.1: When

$$\mu \geq (1+\delta)\Lambda + \frac{\theta_v}{R(1-\delta)} + \frac{\theta_f}{R - T + b},$$

the system behaves as a large one, and $\bar{b}$ is the value of b to make this inequality hold as an equality. Q.E.D.

Proof of Proposition EC.5: It can be verified that

$$(r_f - b)\left(\mu - \frac{\theta_f}{R-T+b}\right)$$

is a concave function of b, and the first order condition leads to the solution

$$\tilde{b}^{NV} = \sqrt{\frac{\theta_f(R-T+r_f)}{\mu}} - R + T.$$

Then, it is straightforward to see that $\tilde{b}^{NV}$ is the optimal b if it falls in $[\underline{b}_{NV},\ \bar{b}_{NV}]$. Otherwise, the optimal b takes the boundary values of $[\underline{b}_{NF},\ \bar{b}_{NF}]$. Q.E.D.

Proof of Proposition EC.6: We first demonstrate the assumption made in the proposition, which is equivalent to $f(0) < 0$.

ASSUMPTION EC.1. *The following inequality holds:*

$$\frac{\delta(\theta_f - \theta_v) + (1-\delta)\sqrt{\delta\theta_f\theta_v}}{R-\delta T} < \frac{\theta_f}{R-T} - \frac{\delta\theta_v}{(1-\delta)R}.$$

Note that the sign of

$$\frac{\delta(\theta_f - \theta_v) + (1-\delta)\sqrt{\delta\theta_f\theta_v}}{(R-\delta T+\delta b)} - \frac{\theta_f}{R-T+b} + \frac{\delta\theta_v}{(1-\delta)R}$$

is as same as the sign of

$$\left(\delta(\theta_f - \theta_v) + (1-\delta)\sqrt{\delta\theta_f\theta_v}\right)(R-T+b) - \theta_f(R-\delta T+\delta b) + \frac{\delta\theta_v(R-\delta T+\delta b)(R-T+b)}{(1-\delta)R}, \quad \text{(EC.66)}$$

as $R-\delta T > 0$ and $R-T+b > 0$. It can be verified that (EC.66) is a quadratic convex function of b. Therefore, if Assumption EC.1 holds, (EC.66) is negative when $b=0$. Thus there exists a unique $b^0 > 0$, such that (EC.66) equals to 0 when $b = b^0$, and when $b \le b^0$,

$$\frac{\delta(\theta_f - \theta_v) + (1-\delta)\sqrt{\delta\theta_f\theta_v}}{R-\delta T+\delta b} \le \frac{\theta_f}{R-T+b} - \frac{\delta\theta_v}{(1-\delta)R},$$

and when $b > b^0$,

$$\frac{\delta(\theta_f - \theta_v) + (1-\delta)\sqrt{\delta\theta_f\theta_v}}{R-\delta T+\delta b} > \frac{\theta_f}{R-T+b} - \frac{\delta\theta_v}{(1-\delta)R}.$$

Let $\lambda_v^{NF}(b)$ denote the effective λ_v^{NF} in equilibrium under optimal capacity allocation with given b. According to (EC.23) in Theorem EC.1, when $b \le b^0$, $\lambda_v^{NF}(b)$ is the first case of (EC.23); when $b > b^0$, $\lambda_v^{NF}(b)$ is the second case of (EC.23). We therefore have

$$\lambda_v^{NF}(b) = \begin{cases} \min\left\{\Lambda, \frac{\mu}{1+\delta} - \frac{(\sqrt{\delta\theta_f}+\sqrt{\theta_v})^2}{(1+\delta)(R-\delta T+\delta b)}\right\}, & \text{if } b \le b^0, \\ \min\left\{\Lambda, \frac{\mu}{1+\delta} - \frac{\theta_f}{(1+\delta)(R-T+b)} - \frac{\theta_v}{(1+\delta)(1-\delta)R}\right\}, & \text{otherwise.} \end{cases} \quad \text{(EC.67)}$$

Note that when $b < \bar{b}_{NF}$, $\lambda_v^{NF}(b)$ cannot hit Λ by the definition of $\bar{b}_{NF}$. Thus

$$\lambda_v^{NF}(b) = \begin{cases} \frac{\mu}{1+\delta} - \frac{(\sqrt{\delta\theta_f}+\sqrt{\theta_v})^2}{(1+\delta)(R-\delta T+\delta b)}, & \text{if } b \le b^0, \\ \frac{\mu}{1+\delta} - \frac{\theta_f}{(1+\delta)(R-T+b)} - \frac{\theta_v}{(1+\delta)(1-\delta)R}, & \text{otherwise.} \end{cases}$$

One can verify that both

$$(r_v - \delta b)\left(\frac{\mu}{1+\delta} - \frac{(\sqrt{\delta\theta_f}+\sqrt{\theta_v})^2}{(1+\delta)(R-\delta T+\delta b)}\right) \tag{EC.68}$$

and

$$(r_v - \delta b)\left(\frac{\mu}{1+\delta} - \frac{\theta_f}{(1+\delta)(R-T+b)} - \frac{\theta_v}{(1+\delta)(1-\delta)R}\right) \tag{EC.69}$$

are concave in b.

Let b_{NF}^1 and b_{NF}^2 denote the maximizer of $(r_v - \delta b)\lambda_v^{NF}(b)$, which takes the forms given in (EC.68) and (EC.69), receptively. Specifically,

$$b_{NF}^1 = \frac{(\sqrt{\delta\theta_f}+\sqrt{\theta_v})\sqrt{R-\delta T+r_v}}{\delta\sqrt{\mu}} - \frac{R-\delta T}{\delta},$$

which comes from taking the first order condition of (EC.68);

$$b_{NF}^2 = \sqrt{\frac{\theta_f(\delta R - \delta T + r_v)}{\delta\mu - \frac{\delta\theta_v}{(1-\delta)R}}} - R + T,$$

which comes from taking the first order condition of (EC.69).

Thus, we have

$$b^{NF*} = \begin{cases} b_{NF}^1, & \text{if } b_{NF}^1 \le b^0 \\ b_{NF}^2, & \text{otherwise.} \end{cases}$$

And it is straightforward to see that b^{NF*} is the optimal b if it falls in $[\underline{b}_{NF}, \ \bar{b}_{NF}]$. Otherwise, the optimal b should take the boundary values of $[\underline{b}_{NF}, \ \bar{b}_{NF}]$. Q.E.D.

Proof of Proposition EC.7: The objective function is

$$(r_f - b)\left(\Lambda - \frac{1}{\delta}\left(\mu - \Lambda - \frac{\theta_v}{(1-\delta)R} - \frac{\theta_f}{R-T+b}\right)\right) + (r_v - \delta b)\frac{1}{\delta}\left(\mu - \Lambda - \frac{\theta_v}{(1-\delta)R} - \frac{\theta_f}{R-T+b}\right).$$

Its first order derivative is

$$\frac{(1-\delta)\mu}{\delta} - \frac{\Lambda}{\delta} - \frac{\theta_v}{\delta R} + \frac{\theta_f(r_v - r_f) - (1-\delta)\theta_f(R-T)}{\delta(R-T+b)^2}. \tag{EC.70}$$

Its second order derivative is

$$-\frac{2\theta_f(r_v - r_f) - (1-\delta)\theta_f(R-T)}{\delta(R-T+b)^3}. \tag{EC.71}$$

If $r_v - r_f - (1-\delta)(R-T) \le 0$, (EC.71) is non-negative. Thus, the objective function is convex. Then the optimal solution must be on the boundary. i.e., either $\underline{b}_{VF}$ or $\bar{b}_{VF}$.

If $r_v - r_f - (1-\delta)(R-T) > 0$, (EC.71) is negative, then the objective function is concave. The first order condition leads to

$$\sqrt{\frac{\theta_f(r_v - r_f) - (1-\delta)\theta_f(R-T)}{-(1-\delta)\mu + \Lambda + \theta_v/R}} - R + T. \tag{EC.72}$$

When $(1-\delta)\mu - \Lambda - \frac{\theta_v}{R} < 0$, then (EC.72) is well-defined. Thus, the optimal b is (EC.72) if it falls in $[\underline{b}_{VF}, \bar{b}_{VF}]$. Otherwise, the optimal b takes the boundary values of $[\underline{b}_{VF}, \ \bar{b}_{VF}]$.

If $r_v - r_f - (1-\delta)(R-T) > 0$ and $(1-\delta)\mu - \Lambda - \frac{\theta_v}{R} \ge 0$, (EC.72) is not well-defined. However, (EC.70) is always positive, indicating that the objective function is increasing in b. Thus the optimal solution is to set b as large as $\bar{b}_{VF}$. Q.E.D.

Proof of Proposition 1: Follows directly from the definition of the thresholds for μ. Q.E.D.

Proof of Proposition 2: The first fact is that, if the original optimal solution without subsidy and the joint optimal solution with subsidy are in the same scenario (region), then the access rate will not decrease. The reason is that, with optimal subsidy, the marginal revenue is decreased but the total revenue is increased. Thus, the access rate will not decrease.

Next, we examine each case.

1. Large system: in this case,

$$\mu \ge (1+\delta)\Lambda + \frac{\theta_f}{R-T} + \frac{\theta_v}{(1-\delta)R}.$$

It is able to achieve Region F and V and obtain the largest revenue $\max\{r_f, r_v\}\Lambda$ without subsidy. Thus, the optimal subsidy is 0, and the total access rate is still Λ.

2. Medium-sized system or small system with $\mu \ge \Lambda + \theta_f[R-T]^{-1}$: in this case,

$$\Lambda + \frac{\theta_f}{R-T} \le \mu < (1+\delta)\Lambda + \frac{\theta_f}{R-T} + \frac{\theta_v}{(1-\delta)R}.$$

- If $r_v \le r_f$, it is able to achieve Region F which generates the largest revenue $r_f\Lambda$ without subsidy. Thus, $b^* = 0$, and the total access rate is still Λ.
- If

$$\frac{r_v}{r_f} \ge (1+\delta)\max\left\{1, \frac{\Lambda}{\mu - \frac{\theta_f}{R-T} - \frac{\theta_v}{(1-\delta)R}}\right\}, \tag{EC.73}$$

then we have $r_v \ge (1+\delta)r_f$ since $\mu - \frac{\theta_f}{R-T} > \mu - \frac{\theta_f}{R-T} - \frac{\theta_v}{(1-\delta)R}$. Per Propositions EC.2 and EC.4, Scenario VF is dominated by Scenario NF for any b. Since Region F, which leads to full access rate Λ, can be achieved without b, then the only one possible situation for decreased access rate is that the original optimal solution is in Region F but the joint optimal solution with subsidy is in Region BV. Note that $r_f\Lambda$ is the revenue achieved in Region F, and

$$r_v\left(\frac{\mu}{1+\delta}-\frac{\theta_f}{(1+\delta)(R-T)}-\frac{\theta_v}{(1+\delta)(1-\delta)R}\right)$$

is the lower bound for the revenue in Region BV. When condition (EC.73) holds, the original optimal without subsidy cannot be in Region F. Thus the access rate will not decrease with subsidy.

3. Small system with $\mu<\Lambda+\theta_f[R-T]^{-1}$: in this case,

$$\max\left\{\frac{\theta_f}{R-T},\underline{\mu}^{BV}\right\}\leq\mu<\Lambda+\frac{\theta_f}{R-T}.$$

- If

$$\frac{r_v}{r_f}\leq\min\left\{1,\frac{(1+\delta)(\mu-\frac{\theta_f}{R-T})}{\mu-\frac{(\sqrt{\delta\theta_f}+\sqrt{\theta_v})^2}{R-\delta T}}\right\},\tag{EC.74}$$

then $r_v\leq r_f$. Per Propositions EC.2, Region BVF is dominated by Region BF for any b. Without subsidy, the revenue in Region BF is $r_f(\mu-\frac{\theta_f}{R-T})$, and

$$r_v\left(\frac{\mu}{1+\delta}-\frac{\left(\sqrt{\delta\theta_f}+\sqrt{\theta_v}\right)^2}{(1+\delta)(R-\delta T)}\right)$$

is the upper bound for the revenue in Region BV. Since (EC.74) holds, the original optimal solution without subsidy is in Region BF. Thus, the only one possible situation for decreased access rate is that the joint optimal solution with subsidy is in Region BV. Suppose that the joint optimal solution with subsidy is in Region BV, we must have $(r_v-b^*)\lambda_v^{BV*}(b^*)\geq r_f\lambda_f^{BF*}(0)$. Since $r_v\leq r_f$, we must have $\lambda_v^{BV*}(b^*)\geq\lambda_f^{BF*}(0)$, indicating non-decreased access rate.

- If (EC.73) holds, i.e.,

$$\frac{r_v}{r_f}\geq\frac{(1+\delta)(\mu-\frac{\theta_f}{R-T})}{\mu-\frac{\theta_f}{R-T}-\frac{\theta_v}{(1-\delta)R}},$$

then we have $r_v\geq(1+\delta)r_f$ since $\mu-\frac{\theta_f}{R-T}>\mu-\frac{\theta_f}{R-T}-\frac{\theta_v}{(1-\delta)R}$. Per Propositions EC.2 and EC.4, Scenario VF is dominated by Scenario NF for any b. Without subsidy, the revenue in Region BF is $r_f(\mu-\frac{\theta_f}{R-T})$, and

$$r_v\left(\frac{\mu}{1+\delta}-\frac{\theta_f}{(1+\delta)(R-T)}-\frac{\theta_v}{(1+\delta)(1-\delta)R}\right)$$

is the lower bound for the revenue in Region BV. Since (EC.73) holds, the original optimal solution without subsidy is in Region BV. Thus, the only one possible situation for decreased access rate is that the joint optimal solution with subsidy is in Region BF. Suppose that the joint optimal solution with subsidy is in Region BF, we must have $(r_f-b^*)\lambda_f^{BF*}(b^*)\geq r_v\lambda_v^{BV*}(0)$. Since $r_v>r_f$, we must have $\lambda_f^{BF*}(b^*)>\lambda_v^{BV*}(0)$, indicating non-decreased access rate. Q.E.D.

Proof of Proposition 3: Following the arguments in the proof of Proposition 2, we know the access rate will not decrease under any of the following conditions.

1. Large system.
2. Medium-sized system or small system with $\mu \geq \Lambda + \theta_f[R-T]^{-1}$, if

$$\frac{r_v}{r_f} \leq 1 \quad \text{or} \quad \frac{r_v}{r_f} \geq (1+\delta)\max\left\{1, \frac{\Lambda}{\mu - \frac{\theta_f}{R-T} - \frac{\theta_v}{(1-\delta)R}}\right\}.$$

3. Small system with $\mu < \Lambda + \theta_f[R-T]^{-1}$, if

$$\frac{r_v}{r_f} \leq \min\left\{1, \frac{(1+\delta)(\mu - \frac{\theta_f}{R-T})}{\mu - \frac{(\sqrt{\delta\theta_f}+\sqrt{\theta_v})^2}{R-\delta T}}\right\} \quad \text{or} \quad \frac{r_v}{r_f} \geq \frac{(1+\delta)(\mu - \frac{\theta_f}{R-T})}{\mu - \frac{\theta_f}{R-T} - \frac{\theta_v}{(1-\delta)R}}.$$

The remaining conditions we have to examine are

1. Medium-sized system or a small system with $\mu \geq \Lambda + \theta_f[R-T]^{-1}$, if

$$1 < \frac{r_v}{r_f} \leq \frac{(1+\delta)\Lambda}{\min\left\{(1+\delta)\Lambda, \mu - \frac{(\sqrt{\delta\theta_f}+\sqrt{\theta_v})^2}{R-\delta T+\delta b}\right\}}.$$

2. Small system with $\mu < \Lambda + \theta_f[R-T]^{-1}$, if

$$1 < \frac{r_v}{r_f} \leq \frac{(1+\delta)\{\Lambda, \mu - \frac{\theta_f}{R-T+b}\}}{\mu - \frac{(\sqrt{\delta\theta_f}+\sqrt{\theta_v})^2}{R-\delta T+\delta b}}.$$

Next, we examine each case.

1. Medium-sized system or small system with $\mu \geq \Lambda + \theta_f[R-T]^{-1}$: in this case,

$$\Lambda + \frac{\theta_f}{R-T} \leq \mu < (1+\delta)\Lambda + \frac{\theta_f}{R-T} + \frac{\theta_v}{(1-\delta)R}.$$

If

$$1 < \frac{r_v}{r_f} \leq \frac{(1+\delta)\Lambda}{\min\left\{(1+\delta)\Lambda, \mu - \frac{(\sqrt{\delta\theta_f}+\sqrt{\theta_v})^2}{R-\delta T+\delta b}\right\}}, \tag{EC.75}$$

then we have $r_v > r_f$. Since (EC.75) holds, then Scenario NF is dominated by Scenario NV after b is utilized. Thus the access rate cannot decrease as Scenario NV and Scenario VF both achieve full access rate.

2. Small system with $\mu < \Lambda + \theta_f[R-T]^{-1}$: in this case,

$$\max\left\{\frac{\theta_f}{R-T}, \underline{\mu}^{BV}\right\} \leq \mu < \Lambda + \frac{\theta_f}{R-T}.$$

If

$$1 < \frac{r_v}{r_f} \leq \frac{(1+\delta)\{\Lambda, \mu - \frac{\theta_f}{R-T+b}\}}{\mu - \frac{(\sqrt{\delta\theta_f}+\sqrt{\theta_v})^2}{R-\delta T+\delta b}}, \tag{EC.76}$$

then we have $r_v > r_f$. Since (EC.76) holds, then Scenario NF is dominated by Scenario NV after b is utilized. Thus the access rate cannot decrease as: 1) the original optimal capacity allocation without b must be in Scenario NF or NV; 2) after b is utilized, Scenario VF achieves full access rate; 3) if the original optimal capacity allocation after b is utilized is in Scenario NF, then the access rate must be non-decreasing since $r_v > r_f$. Q.E.D.

Proof of Proposition 4: We first show that, for any given set of model parameters, there exists a government-funded transportation subsidy under which the total access rate is not smaller than that achieved under a provider-funded transportation subsidy. Consider the following optimization problem for the government:

$$\begin{aligned}\max_{b\geq 0}\ & \lambda_f(\mu_f,\mu_v,b)+\lambda_v(\mu_f,\mu_v,b) \\ \text{s.t. }\ & (\mu_f,\mu_v) \text{ solves (P2)}, \\ & \lambda_f(\mu_f,\mu_v,b) \text{ and } \lambda_v(\mu_f,\mu_v,b) \text{ are defined in Theorem EC.1.}\end{aligned} \tag{EC.P3}$$

1. When $r_f \geq r_v$, according to Corollary EC.1, the government can set b as large as possible to achieve the maximum access rate $\min\{\Lambda, \mu-\theta_f[R-T+\bar{b}]^{-1}\}$, where $\bar{b}$ is the maximum b the government can set which is large enough. Thus there exists a government-funded transportation subsidy under which the total access rate is not less than that achieved under a provider-funded transportation subsidy.
2. When $r_f < r_v$, we construct the following problem.

$$\begin{aligned}\max_{b\geq 0}\ & \lambda_f(\mu_f,\mu_v,b)+\lambda_v(\mu_f,\mu_v,b) \\ \text{s.t. }\ & (\mu_f,\mu_v) \text{ solves (P1) for the given } b, \\ & \lambda_f(\mu_f,\mu_v,b) \text{ and } \lambda_v(\mu_f,\mu_v,b) \text{ are defined in Theorem EC.1.}\end{aligned} \tag{EC.P4}$$

We first show that the optimal objective value of (EC.P3) is no smaller than that of (EC.P4).

To see this, for any given b, let $(\lambda_{f,1},\lambda_{v,1})$ denote the corresponding access rate when (μ_f,μ_v) solves (P2); let $(\lambda_{f,2},\lambda_{v,2})$ denote the corresponding access rate when (μ_f,μ_v) solves (P1) (for the given b). We must have $r_f\lambda_{f,1}+r_v\lambda_{v,1} \geq r_f\lambda_{f,2}+r_v\lambda_{v,2}$ and $(r_f-b)\lambda_{f,1}+(r_v-\delta b)\lambda_{v,1} \leq (r_f-b)\lambda_{f,2}+(r_v-\delta b)\lambda_{v,2}$. Thus $\lambda_{f,1}+\delta\lambda_{v,1} \geq \lambda_{f,2}+\delta\lambda_{v,2}$. According to Corollary EC.1, it is easy to see that:

- when $\lambda_{f,1} > 0$ and $\lambda_{v,1} > 0$, then $\lambda_{f,1}+\lambda_{v,1} = \Lambda$, thus $\lambda_{f,1}+\lambda_{v,1} \geq \lambda_{f,2}+\lambda_{v,2}$;
- when $\lambda_{v,1} = 0$ and $\lambda_{v,2} = 0$, or $\lambda_{f,1} = 0$ and $\lambda_{f,2} = 0$,then $\lambda_{f,1}+\lambda_{v,1} \geq \lambda_{f,2}+\lambda_{v,2}$;
- when $\lambda_{f,1} = 0$ and $\lambda_{v,2} = 0$, then $\delta\lambda_{v,1} \geq \lambda_{f,2}$, thus $\lambda_{f,1}+\lambda_{v,1} \geq \lambda_{f,2}+\lambda_{v,2}$;
- when $\lambda_{v,1} = 0$ and $\lambda_{f,2} = 0$, then $r_f\lambda_{f,1} \geq r_v\lambda_{v,2}$. Since $r_f < r_v$, then $\lambda_{f,1}+\lambda_{v,1} \geq \lambda_{f,2}+\lambda_{v,2}$.

Consequently, for any given b, the objective value of (EC.P3) is no smaller than that of (EC.P4); thus the optimal objective value of (EC.P3) is no smaller than that of (EC.P4).

Let (μ_f^*, μ_v^*, b^*) denote the optimal solution to (P1). It is easy to see (μ_f^*, μ_v^*, b^*) is a feasible solution to (EC.P4). Since (EC.P3) dominates (EC.P4), then there exists a government-funded transportation subsidy under which the total access rate is not less than that achieved under a provider-funded transportation subsidy when $r_f < r_v$.

Now we show that, when μ, r_f and r_v exceed certain thresholds, the provider-funded transportation subsidy can attain the maximum access rate.

When $\mu > (1+\delta)\Lambda$, Corollary EC.1 and Proposition 1 imply that the total access rate under Scenario VF attains its maximum value of Λ. Hence, the total access rate will also reach this maximum Λ if the provider-funded transportation subsidy b^* attains the highest feasible value of b under Scenarios NF and NV. To ensure this, Propositions EC.5 and EC.6 require that $\tilde{b}_{NV} \geq \bar{b}_{NV}$ and $\tilde{b}_{NF} \geq \bar{b}_{NF}$. It can be verified that these inequalities hold whenever

$$r_f \geq \frac{\mu\theta_f}{(\mu-\Lambda)^2} - R + T,$$

and

$$r_v \geq \max\left\{ \frac{\mu(\sqrt{\delta\theta_f}+\sqrt{\theta_v})^2}{(\mu-(1+\delta)\Lambda)^2} - R + \delta T,\ \left(\frac{\delta\mu}{\theta_f} - \frac{\delta\theta_v}{\theta_f(1-\delta)R}\right)\left[\frac{(\sqrt{\delta\theta_f}+\sqrt{\theta_v})^2}{\delta[\mu-(1+\delta)\Lambda]} - \frac{(1-\delta)R}{\delta}\right]^2 - \delta(R-T) \right\}.$$

Q.E.D.

EC.3.4 Proofs of Analytical Results in E-Companion EC.2

Proof of Lemma EC.2: It directly follows the definitions of $U_v^t(\lambda_f,\lambda_v)$, $U_f^t(\lambda_f,\lambda_v)$ and $U_b^t(\lambda_f,\lambda_v)$. Q.E.D.

Proof of Theorem EC.2: We first show that the seven regions of (μ_f,μ_v) defined in Definition EC.3 are mutually exclusive and collectively exhaustive. By Definition EC.3, it is easy to see (EC.33) v.s. (EC.47) divide the (μ_f,μ_v) space; (EC.34) v.s. (EC.50) divide the (μ_f,μ_v) space; (EC.35) v.s. (EC.56) divide the (μ_f,μ_v) space; (EC.36) with (EC.37) and (EC.38) v.s. (EC.51) divide the (μ_f,μ_v) space; (EC.39) v.s. (EC.48) divide the (μ_f,μ_v) space; (EC.40) with $\mu_f > \Lambda$ (implied by (EC.39)) v.s. (EC.57) divide the (μ_f,μ_v) space; (EC.41) v.s. (EC.49) divide the (μ_f,μ_v) space; (EC.42) v.s. (EC.58) with (EC.59) divide the (μ_f,μ_v) space; (EC.43) with (EC.44) v.s. (EC.52) with (EC.53) divide the (μ_f,μ_v) space; (EC.45) with (EC.46) v.s. (EC.54) with (EC.55) divide the (μ_f,μ_v) space. Thus the seven regions of (μ_f,μ_v) defined in Definition EC.3 are mutually exclusive and collectively exhaustive.

In Region B, everyone chooses balking, i.e., $\lambda_f = 0$, and $\lambda_v = 0$. We should have $U_f^t(0,0) \leq 0$ and $U_v^t(0,0) \leq 0$, for $\forall\, t$. By Lemma EC.2, we arrive at

$$U_f^0(0,0) \leq 0$$

and

$$U_v^0(0,0) \leq 0,$$

which yield (EC.33) and (EC.34), indicating that if (EC.33) and (EC.34) hold then we must have $U_f^t(0,0) \leq 0$ and $U_v^t(0,0) \leq 0$, for $\forall\, t$.

In Region V, everyone chooses virtual channel, i.e., $\lambda_f = 0$, and $\lambda_v = \Lambda$. Then we have $U_v^t(0,\Lambda) \geq U_f^t(0,\Lambda)$ and $U_v^t(0,\Lambda) \geq 0$, for $\forall\, t$. By Lemma EC.2, we must have

$$U_v^0(0,\Lambda) \geq U_f^0(0,\Lambda)$$

and

$$U_v^1(0,\Lambda) \geq 0,$$

which yield (EC.35) and (EC.36) with (EC.37) and (EC.38), indicating that if (EC.35), (EC.36), (EC.37) and (EC.38) hold then we must have $U_v^t(0,\Lambda) \geq U_f^t(0,\Lambda)$ and $U_v^t(0,\Lambda) \geq 0$, for $\forall\, t$.

In Region F, everyone chooses face-to-face channel, i.e., $\lambda_f = \Lambda$, and $\lambda_v = 0$. Then we have $U_f^t(\Lambda,0) \geq U_v^t(\Lambda,0)$ and $U_f^t(\Lambda,0) \geq 0$, for $\forall\, t$. By Lemma EC.2, we must have

$$U_f^1(\Lambda,0) \geq U_v^1(\Lambda,0)$$

and

$$U_f^1(\Lambda,0) \geq 0,$$

which yield (EC.39) and (EC.40) with $\mu_f > \Lambda$ (which is implied by (EC.39)), indicating that if (EC.39) and (EC.40) hold then we must have $U_f^t(\Lambda,0) \geq U_v^t(\Lambda,0)$ and $U_f^t(\Lambda,0) \geq 0$, for $\forall\, t$.

Denote $1/(\mu_f - \lambda_f - \delta\lambda_v)$ as w_f and $1/(\mu_v - \lambda_v)$ as w_v. Define three thresholds:

$$t_{fb} = R - \theta_f w_f, \tag{EC.77}$$

$$t_{vb} = \frac{R - \theta_v w_v}{\delta} - \theta_f w_f, \tag{EC.78}$$

$$t_{fv} = \frac{\theta_v w_v}{1-\delta} - \theta_f w_f. \tag{EC.79}$$

By their definitions, we have if $t < t_{fb}$ then $U_f > U_b$, and if $t > t_{fb}$ then $U_f < U_b$; if $t < t_{vb}$ then $U_v > U_b$, and if $t > t_{vb}$ then $U_v < U_b$; if $t < t_{fv}$ then $U_f > U_v$, and if $t > t_{fv}$ then $U_f < U_v$.

In Region BVF, some patients choose face-to-face channel, some choose virtual channel and some choose balking, i.e., $\lambda_f > 0$, $\lambda_v > 0$ and $\lambda_f + \lambda_b < \Lambda$. Then we have $U_f^t(\lambda_f,\lambda_v) \geq \max\{U_v^t(\lambda_f,\lambda_v), 0\}$ for some

t, $U_v^t(\lambda_f,\lambda_v) \geq \max\{U_f^t(\lambda_f,\lambda_v),0\}$ for some t, and $0 \geq \max\{U_f^t(\lambda_f,\lambda_v),U_v^t(\lambda_f,\lambda_v)$ for some t. By Lemma EC.2, we must have $0 < t_{fv} < t_{vb} < 1$. Then,

$$t_{fv} = \frac{\lambda_f}{\Lambda} \quad \text{and} \quad t_{vb} = \frac{\lambda_f + \lambda_v}{\Lambda}.$$

We then have

$$t_{vb} - t_{fv} = \frac{R}{\delta} - \frac{\theta_v}{\delta(1-\delta)(\mu_v - \lambda_v)} = \frac{\lambda_v}{\Lambda}.$$

$\lambda_v > 0$ is equivalent to

$$\max_{\lambda_v > 0} \left\{ \frac{R}{\delta} - \frac{\theta_v}{\delta(1-\delta)(\mu_v - \lambda_v)} - \frac{\lambda_v}{\Lambda} \right\} > 0,$$

thus we can have (EC.41).

We can also have

$$t_{vb} = \frac{R}{\delta} - \frac{\theta_v}{\delta(\mu_v - \lambda_v)} - \frac{\theta_f}{\mu_f - \lambda_f - \delta\lambda_v} = \frac{\lambda_f + \lambda_v}{\Lambda}.$$

$\lambda_f < \Lambda - \lambda_v$ is equivalent to

$$\min_{\lambda_f < \Lambda - \lambda_v} \left\{ \frac{R}{\delta} - \frac{\theta_v}{\delta(\mu_v - \lambda_v)} - \frac{\theta_f}{\mu_f - \lambda_f - \delta\lambda_v} - \frac{\lambda_f + \lambda_v}{\Lambda} \right\} < 0,$$

for any $0 < \lambda_v < \Lambda$ and when $\mu_f - \lambda_f - \delta\lambda_v > 0$. It is equivalent to

$$\max_{0 < \lambda_v < \Lambda} \left\{ \frac{R}{\delta} - \frac{\theta_v}{\delta(\mu_v - \lambda_v)} - \frac{\theta_f}{\mu_f - \Lambda + (1-\delta)\lambda_v} - 1 \right\} < 0, \tag{EC.80}$$

when $\mu_f - \Lambda + (1-\delta)\lambda_v > 0$. One can show (EC.42) is equivalent to (EC.80) by taking the maximization of the LHS of (EC.80).

We can also have

$$t_{fv} = \frac{\theta_v}{(1-\delta)(\mu_v - \lambda_v)} - \frac{\theta_f}{\mu_f - \lambda_f - \delta\lambda_v} = \frac{\lambda_f}{\Lambda}.$$

$\lambda_f > 0$ is equivalent to

$$\max_{\lambda_f > 0} \left\{ \frac{\theta_v}{(1-\delta)(\mu_v - \lambda_v)} - \frac{\theta_f}{\mu_f - \lambda_f - \delta\lambda_v} - \frac{\lambda_f}{\Lambda} \right\} > 0,$$

for any $0 < \lambda_v < \Lambda$. It is equivalent to

$$\frac{\theta_v}{(1-\delta)(\mu_v - \lambda_v)} - \frac{\theta_f}{\mu_f - \delta\lambda_v} > 0, \tag{EC.81}$$

for any $0 < \lambda_v < \Lambda$. Thus we have if $(1-\delta)\theta_f - \delta\theta_v > 0$ then

$$\lambda_v > \frac{(1-\delta)\theta_f\mu_v - \theta_v\mu_f}{(1-\delta)\theta_f - \delta\theta_v},$$

and if $(1-\delta)\theta_f - \delta\theta_v < 0$ then

$$\lambda_v < \frac{(1-\delta)\theta_f\mu_v - \theta_v\mu_f}{(1-\delta)\theta_f - \delta\theta_v}.$$

We first consider the case of $(1-\delta)\theta_f - \delta\theta_v > 0$. In this case, if $(1-\delta)\theta_f\mu_v - \theta_v\mu_f < 0$ then (EC.81) holds; if $(1-\delta)\theta_f\mu_v - \theta_v\mu_f \geq 0$ then we have

$$\max_{\lambda_v > \frac{(1-\delta)\theta_f\mu_v - \theta_v\mu_f}{(1-\delta)\theta_f - \delta\theta_v}} \left\{ \frac{R}{\delta} - \frac{\theta_v}{\delta(1-\delta)(\mu_v - \lambda_v)} - \frac{\lambda_v}{\Lambda} \right\} > 0.$$

One can show (EC.43) with (EC.44) is equivalent to it.

We then consider the case of $(1-\delta)\theta_f - \delta\theta_v < 0$. In this case, we must have $(1-\delta)\theta_f\mu_v - \theta_v\mu_f < 0$ and

$$\min_{\lambda_v < \frac{(1-\delta)\theta_f\mu_v - \theta_v\mu_f}{(1-\delta)\theta_f - \delta\theta_v}} \left\{ \frac{R}{\delta} - \frac{\theta_v}{\delta(1-\delta)(\mu_v - \lambda_v)} - \frac{\lambda_v}{\Lambda} \right\} < 0.$$

One can show (EC.45) with (EC.46) is equivalent to it with $(1-\delta)\theta_f\mu_v - \theta_v\mu_f < 0$.

Now we are ready to derive λ_f and λ_v. Combining

$$t_{fv} = \frac{\lambda_f}{\Lambda}$$

and

$$t_{vb} = \frac{\lambda_f + \lambda_v}{\Lambda}$$

together with (EC.78) and (EC.79), we can solve the equations and obtain

$$\lambda_f = \tfrac{1}{2}\left[\mu_f - \delta\mu_v - \sqrt{(R\Lambda - \mu_f)^2 + 4\theta_f\Lambda} + \delta\sqrt{\left(\frac{R\Lambda}{\delta} - \mu_v\right)^2 + \frac{4\theta_v\Lambda}{\delta(1-\delta)}}\right] \tag{EC.82}$$

$$\lambda_v = \tfrac{1}{2}\left[\frac{R\Lambda}{\delta} + \mu_v - \sqrt{\left(\frac{R\Lambda}{\delta} - \mu_v\right)^2 + \frac{4\theta_v\Lambda}{\delta(1-\delta)}}\right]. \tag{EC.83}$$

In Region BF, some patients choose face-to-face channel and some choose balking, i.e., $0 < \lambda_f < \Lambda$, and $\lambda_v = 0$. Then we have $U_f^t(\lambda_f, 0) \geq \max\{U_v^t(\lambda_f, 0), 0\}$ for some t, and $0 \geq \max\{U_f^t(\lambda_f, 0), U_v^t(\lambda_f, 0)\}$ for some t. By Lemma EC.2, we must have $0 < t_{fb} < 1$ and $t_{vb} \leq t_{fv}$. Then

$$t_{fb} = \frac{\lambda_f}{\Lambda}.$$

Then we have

$$t_{fb} = R - \frac{\theta_f}{\mu_f - \lambda_f} = \frac{\lambda_f}{\Lambda}.$$

Note that $\lambda_f > 0$ is equivalent to

$$\max_{\lambda_f > 0} \left\{ R - \frac{\theta_f}{\mu_f - \lambda_f} - \frac{\lambda_f}{\Lambda} \right\} > 0.$$

thus we can have (EC.47).

Note that $\lambda_f < \Lambda$ is equivalent to

$$\min_{\lambda_f<\Lambda}\left\{R-\frac{\theta_f}{\mu_f-\lambda_f}-\frac{\lambda_f}{\Lambda}\right\}<0.$$

Thus we can have (EC.48).

Note that $\lambda_v = 0$ is equivalent to $t_{vb} \le t_{fv}$, i.e.,

$$\min_{\lambda_v\ge 0}\left\{R-\frac{\theta_v}{(1-\delta)(\mu_v-\lambda_v)}\right\}\le 0.$$

Thus we can have (EC.49).

Now we are ready to derive λ_f. By

$$t_{fb}=R-\frac{\theta_f}{\mu_f-\lambda_f}=\frac{\lambda_f}{\Lambda}$$

and (EC.77), we can solve the equation and obtain

$$\lambda_f=\tfrac{1}{2}\left[R\Lambda+\mu_f-\sqrt{(R\Lambda-\mu_f)^2+4\theta_f\Lambda}\right]. \tag{EC.84}$$

In Region BV, some patients choose virtual channel and some choose balking, i.e., $0<\lambda_v<\Lambda$, and $\lambda_f=0$. Then we have $U_v^t(0,\lambda_v)\ge\max\{U_f^t(0,\lambda_v),0\}$ for some t, and $0\ge\max\{U_f^t(0,\lambda_v),U_v^t(0,\lambda_v)$ for some t. By Lemma EC.2, we must have $t_{fv}\le 0<t_{vb}<1$. Then

$$t_{vb}=\frac{\lambda_v}{\Lambda}.$$

Note that $\lambda_v>0$ is equivalent to

$$\max_{\lambda_v>0}\left\{\frac{R}{\delta}-\frac{\theta_v}{\delta(\mu_v-\lambda_v)}-\frac{\theta_f}{\mu_f-\delta\lambda_v}-\frac{\lambda_v}{\Lambda}\right\}>0.$$

thus we can have (EC.50).

Note that $\lambda_v<\Lambda$ is equivalent to

$$\min_{\lambda_v<\Lambda}\left\{\frac{R}{\delta}-\frac{\theta_v}{\delta(\mu_v-\lambda_v)}-\frac{\theta_f}{\mu_f-\delta\lambda_v}-\frac{\lambda_v}{\Lambda}\right\}<0.$$

thus we can have (EC.51).

Note that $\lambda_f=0$ is equivalent to $t_{fv}\le 0$, i.e.,

$$\min_{\lambda_f\ge 0}\left\{\frac{\theta_v}{(1-\delta)(\mu_v-\lambda_v)}-\frac{\theta_f}{\mu_f-\lambda_f-\delta\lambda_v}\right\}\le 0.$$

Thus we can have

$$\frac{\theta_v}{(1-\delta)(\mu_v-\lambda_v)}-\frac{\theta_f}{\mu_f-\delta\lambda_v}\le 0, \tag{EC.85}$$

for any $\lambda_v \in [0,\Lambda]$. Then we have, if $(1-\delta)\theta_f - \delta\theta_v > 0$ then

$$\lambda_v \leq \frac{(1-\delta)\theta_f\mu_v - \theta_v\mu_f}{(1-\delta)\theta_f - \delta\theta_v},$$

and if $(1-\delta)\theta_f - \delta\theta_v < 0$ then

$$\lambda_v \geq \frac{(1-\delta)\theta_f\mu_v - \theta_v\mu_f}{(1-\delta)\theta_f - \delta\theta_v}.$$

We first consider the case of $(1-\delta)\theta_f - \delta\theta_v > 0$. In this case, we must have $(1-\delta)\theta_f\mu_v - \theta_v\mu_f \geq 0$ and

$$\min_{\lambda_v \leq \frac{(1-\delta)\theta_f\mu_v - \theta_v\mu_f}{(1-\delta)\theta_f - \delta\theta_v}} \left\{ \frac{R}{\delta} - \frac{\theta_v}{\delta(\mu_v - \lambda_v)} - \frac{\theta_f}{\mu_f - \delta\lambda_v} - \frac{\lambda_v}{\Lambda} \right\} \leq 0.$$

One can show (EC.52) with (EC.53) is equivalent to it with $(1-\delta)\theta_f\mu_v - \theta_v\mu_f \geq 0$.

We then consider the case of $(1-\delta)\theta_f - \delta\theta_v < 0$. In this case, if $(1-\delta)\theta_f\mu_v - \theta_v\mu_f < 0$ then (EC.85) holds; if $(1-\delta)\theta_f\mu_v - \theta_v\mu_f \geq 0$ then we have

$$\max_{\lambda_v \geq \frac{(1-\delta)\theta_f\mu_v - \theta_v\mu_f}{(1-\delta)\theta_f - \delta\theta_v}} \left\{ \frac{R}{\delta} - \frac{\theta_v}{\delta(\mu_v - \lambda_v)} - \frac{\theta_f}{\mu_f - \delta\lambda_v} - \frac{\lambda_v}{\Lambda} \right\} \geq 0.$$

One can show (EC.54) with (EC.55) is equivalent to it.

Now we are ready to derive λ_v. By

$$t_{vb} = \frac{\lambda_v}{\Lambda}$$

and (EC.78), we can solve the following equation and obtain λ_v:

$$\frac{R\Lambda}{\delta} - \frac{\theta_v\Lambda}{\delta(\mu_v - \lambda_v)} - \frac{\theta_f\Lambda}{\mu_f - \delta\lambda_v} = \lambda_v \tag{EC.86}$$

In Region VF, some patients choose face-to-face channel and some choose virtual channel, i.e., $\lambda_f > 0$, $\lambda_v > 0$ and $\lambda_f + \lambda_v = \Lambda$. Then we have $U_f^t(\Lambda - \lambda_v, \lambda_v) \geq \max\{U_v^t(\Lambda - \lambda_v, \lambda_v), 0\}$ for some t, and $U_v^t(\Lambda - \lambda_v, \lambda_v) \geq \max\{U_f^t(\Lambda - \lambda_v, \lambda_v), 0\}$ for some t. By Lemma EC.2, we must have $0 < t_{fv} < 1 \leq t_{vb}$. Then

$$t_{fv} = \frac{\Lambda - \lambda_v}{\Lambda}.$$

We first have $\lambda_f = \Lambda - \lambda_v$.

Note that $\lambda_v < \Lambda$ is equivalent to

$$\max_{\lambda_v < \Lambda} \left\{ \frac{\theta_v}{(1-\delta)(\mu_v - \lambda_v)} - \frac{\theta_f}{\mu_f - \Lambda + (1-\delta)\lambda_v} - \frac{\Lambda - \lambda_v}{\Lambda} \right\} > 0.$$

Thus we have (EC.56).

Note that $\lambda_v > 0$ is equivalent to

$$\min_{\lambda_v > 0} \left\{ \frac{\theta_v}{(1-\delta)(\mu_v - \lambda_v)} - \frac{\theta_f}{\mu_f - \Lambda + (1-\delta)\lambda_v} - \frac{\Lambda - \lambda_v}{\Lambda} \right\} < 0.$$

Thus we have (EC.57).

Note that $\lambda_b = 0$ is equivalent to $t_{vb} \geq 1$, i.e.,

$$\max_{0<\lambda_v<\Lambda} \left\{ \frac{R}{\delta} - \frac{\theta_v}{\delta(\mu_v - \lambda_v)} - \frac{\theta_f}{\mu_f - \Lambda + (1-\delta)\lambda_v} - 1 \right\} \geq 0, \tag{EC.87}$$

and $\mu_f - \Lambda + (1-\delta)\lambda_v > 0$. One can show (EC.58) with (EC.59) is equivalent to (EC.87) with $\mu_f - \Lambda + (1-\delta)\lambda_v > 0$ by taking the maximization of the LHS of (EC.87).

By

$$t_{fv} = \frac{\Lambda - \lambda_v}{\Lambda}$$

and (EC.79), we can solve the following equation and obtain λ_v:

$$\Lambda - \frac{\theta_v \Lambda}{(1-\delta)(\mu_v - \lambda_v)} + \frac{\theta_f \Lambda}{\mu_f - \Lambda + (1-\delta)\lambda_v} = \lambda_v. \tag{EC.88}$$

Q.E.D.